Colby

MA121: Fall 2018

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Midterm 1 Overview and Review

This Midterm 1 Overview and Review serves three purposes:

- To provide an overview of the format of Midterm 1.
- To provide a detailed list of topics you are expected to know for Midterm 1 and can expect to be tested on.
- To provide some practice problems to aid your studying.

Exam Format

(I) Unless otherwise arranged, you have two hours to complete the exam.

(II) Calculators are **not permitted** for use during the exam.

(III) The exam is **closed book**: notes, textbooks, computers, mobile devices, listening devices are **not permitted** for use during the exam.

(IV) There will be a total of five problems on the exam. To receive full credit you will need to provide correct and complete solutions to all five problems.

(V) There will be one problem consisting of several True/False subproblems.

(VI) There will be one short-answer problem consisting of several short, computational subproblems.

(VII) There will be three long-answer problems. Each long-answer problem may have multiple parts.

(VIII) A student with a solid grasp of the material and excellent problem-solving ability should be able to complete the exam in < 1 hour.

Outline of Topics to Know

Theory

Except for the ϵ-δ definition of the limit, you should know all definitions/concepts from lecture (and used in homework) precisely. In particular, you should know the definitions of the following terms/concepts and how they are used:

(a) The definition of a function, its domain and range.

(b) The different ways we can represent a function.

(c) What it means for a function to be non-increasing/non-decreasing/increasing/decreasing.

(d) What it means to compose two function $f(x)$, $g(x)$.

(e) The definition of the inverse function, when it exists, and its relationship to the original function.

(f) The definition of polynomial, rational, exponential, logarithmic functions and the standard trigonometric functions and their inverses.

(g) The definition of secant/tangent lines for a graph of a function.

(h) The definition of the limit; one-sided and two-sided limits; limits at infinity.

(i) The definition of continuity at a point.

(j) What it means for a function to be continuous everywhere.

(k) The definition of the derivative at a point.

(l) The definition of the derivative function.

(m) Leibniz and Lagrange notation for derivatives.

(n) The definition of the second derivative.

(o) The definition of Euler's number e.

In addition, **you should know examples of functions for which certain limits/derivatives do not exist**.

Major Results

You should thoroughly understand all of the major results we've discussed this semester; this includes all of the theorems and propositions we've discussed in class. This means, in particular, having a solid understanding of each theorem's hypothesis and, if I have discussed the necessity of a certain hypotheses, you should understand why. Here are some of the big results we've discussed so far.

(i) You should know the (algebraic) laws of limits and how to use them.

(ii) You should know that all 'nice' functions are continuous everywhere on their domain.

(iii) You should know how to use continuous functions to compute limits.

(iv) You should know the (algebraic) laws for continuous functions.

(v) You should know the Intermediate Value Theorem and how to use it to approximate solutions to an equation.

(vi) You should know the relationship between the derivative at a point and the slope of the tangent line.

(vii) You should know the relationship between the sign of the derivative at a point and the behaviour of the function near to that point.

(viii) You should know the Power Law, Linearity, the Product Rule, the Quotient Rule aka the **Derivative Rules**.

(ix) You should know the derivatives of $\sin(x)$, $\cos(x)$, $\tan(x)$.

(x) You should know the derivatives of exponential functions.

(xi) You should know the relationship between the sign of the second derivative and concavity.

Computation

In general, you should know how to work problems connected to each key concept discussed in the previous section. As I said before, if you understand all of the lecture material and all of the homework (both the "turn in" and "do not turn in" problems), are able to work quickly and efficiently, you should perform well on the exam. Here is an incomplete list of things we have done so far this semester:

1. You should know that ∞ is not a number and never should be considered as an input to a function.
2. Determining a formula for the inverse function.
3. Computing limits of functions using Limit Laws.
4. Computing limits of functions using continuity.
5. Computing the derivative at a point using the limit definition.
6. Computing the derivative function using the limit definition.
7. Using the Derivative Rules to compute derivative functions.
8. Computing the equation of tangent lines to graphs.

Review Problems

The following problems will provide you with good practice for the midterm. If you want more problems then please feel free to ask.

Chapter 1 Summary (*p.68-73*) 11-13, 20-21, 44-46
Chapter 2 Summary (*p.116-121*) 11, 13-18, 20-21, 22-24, 26-30, 31-35, 36, 38, 45, 50, 51, 52, 60-61, 63 (*Use* `desmos.com` *for graphing purposes*)
Chapter 3 Summary (*p.180-183*) 2-6, 8, 10, 13-14, 21-24, 43-45, 49-52, 57-58, 60-62, 65, 68, 71-73, 79-81 (*Note: you will find it useful to recall that* $e^{2x} = e^x \cdot e^x$)