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Practice Midterm 1

Disclaimer: This Practice Midterm consists of problems of a similar difficulty as will be on the actual midterm. However, **problems on the actual midterm may or may not be quite different in nature, and may or may not focus on different course material**. However, the actual midterm will have a similar format: one true/false problem, one short-answer problem, three long-answer problems.

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1. True/False (no justification required)
   (a) If $\lim_{x\to a} f(x)$ exists then $f'(a)$ exists.
   (b) If $\arcsin(x) = 5$ then $x = \sin(5)$.
   (c) If $f(5) = -1$ then $\lim_{x\to 5} f(x) = -1$.
   (d) $\frac{d}{dx}x^e = x^e$.
   (e) Let $f(x), g(x)$ be functions that are continuous at $x = 1$. If $g(1) = 1$ and $f(1) = -1$ then $\lim_{x\to 1} f(g(x)) = -1$.

2. Compute $f'(x)$ for the given function $f(x)$.
   (a) $f(x) = (5x^2 + 2\sqrt[3]{x^2})e^x + \pi^e \sin(x)$
   (b) $f(x) = \frac{x^{12/5} - 3x + 10}{e^x + e^{2x}}$

3. Let k, l be constants.
   (a) Using limit laws, or otherwise, determine
   $$\lim_{x\to 1} \frac{x^2 + kx - 2}{x^2 + 5x - 5}$$
   Note: your answer will be in terms of k.
   (b) Define
   $$f(x) = \begin{cases} \frac{x^2+kx-2}{x^2+5x-5}, & x \neq 1 \\ l, & x = 1 \end{cases}$$
   Determine all values of k, l so that $f(x)$ is continuous at $x = 1$. Be careful to justify why you know that $f(x)$ will continuous at $x = 1$ for these values of k, l.

4. Let $f(x) = \sqrt{2 - x}$.
   (a) What is the domain of $f(x)$?
   (b) Use the limit definition of the derivative to show that $f'(1) = -1/2$.
   (c) Determine the tangent line to the graph $y = f(x)$ at $(1, f(1))$.

5. (a) Let $h(t)$ be a function with derivative $h'(t)$. Show that
   $$\frac{d}{dt}(h(t))^2 = 2h'(t)h(t)$$
   (b) Use (a) to compute $\frac{d}{dt}\sqrt{e^t - t^2}$.