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Practice Final

Disclaimer: This Practice Midterm consists of problems of a similar difficulty as will be on the actual midterm. However, **problems on the actual midterm may or may not be quite different in nature, and may or may not focus on different course material**. However, the actual midterm will have a similar format: one true/false problem, one short-answer problem, three long-answer problems.

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1. True/False (no justification required)

   (a) $\lim_{h \to 0} \dfrac{e^{1+h} - e}{h} = e$

   (b) If $f''(x) > 0$ then $f(x)$ has a local minimum.

   (c) The limit $\lim_{x \to 0} \dfrac{\sin(x)}{x}$ does not exist.

   (d) The local linearisation of $f(x) = \sqrt{x}$ at $x = 1$ is $L(x) = 1 + \frac{x}{2}$

   (e) The function $f(x) = |x|$ does not possess an antiderivative.

   (f) If $f'(x) < 1$, for all $0 \leq x \leq 10$, then $f(x) \leq x$, for all $0 \leq x \leq 10$.

   (g) The formula $\int_0^x f''(x)dx = f'(x) - f'(0)$ holds.

   (h) If $F(x)$ is an antiderivative of $f(x)$ then $F'(x) = f(x)$.

   (i) $\int e^{x^2} dx = \dfrac{e^{x^2}}{2x} + C$

   (j) The Mean Value Theorem states that if you drive from Waterville, ME, to Portland, ME, at 70 miles per hour, on average, then there is a moment during the drive when you were travelling at exactly 70 miles per hour.

2. Compute $f'(x)$ for the given function $f(x)$.

   (a) $f(x) = \ln(2x+1)\cos(e^x)$

   (b) $f(x) = \int_1^{x^2} \sin(t^2)dt$

3. Compute the following integral problems.

   (a) $\int \dfrac{1}{x\ln(x)}dx$

   (b) $\int_0^1 3x^2 \sqrt[3]{2x^3+8}dx$

4. Let $f(x) = 1 - x$, $g(x) = 1 - x^2$.

   (a) Determine the area bounded between the curves $y = f(x)$ and $y = g(x)$.

   (b) Compute the equation of the tangent line to $y = g(x)$ at $x = 1/2$.

5. Consider the sum

$$\sum_{i=1}^{n} \frac{\pi \sin(\pi i/2n)}{2n}$$

(a) Determine a function $f(x)$ and an interval $a \le x \le b$ so that the above sum is the corresponding right-hand sum R_n of $f(x)$ on $a \le x \le b$.

(b) Using the First Fundamental Theorem of Calculus, compute

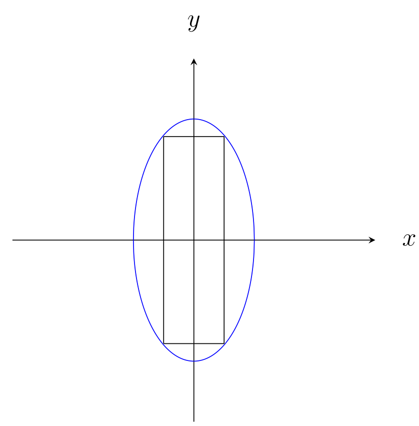
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi \sin(\pi i/2n)}{2n}$$

6. (a) Show that the area A of a rectangle having width $2x$ inscribed inside the ellipse

$$E : \qquad x^2 + \frac{y^2}{4} = 1$$

is

$$A = 4x\sqrt{4 - 4x^2}$$



(b) Explain what's wrong with the following statement: the area of a rectangle having width $2x$ inscribed in E is

$$\int_{-1}^{1} 4x\sqrt{4 - 4x^2}\,dx$$

(c) Determine the maximal area of a rectangle inscribed in the ellipse E

7. Let $f(x) = 1/\sqrt{x-1}$, defined when $x > 1$.

(a) Let $1 < t < 5$. Compute $A(t) = \displaystyle\int_t^5 f(x)dx$.

(b) Compute $\lim_{t\to 1^+} A(t)$.

(c) True/False: the area bounded below $y = f(x)$, $1 < x \le 5$, is $\lim_{t\to 1^+} A(t)$. **Justify your answer**.