Colby

Math 434, Representation Theory
Spring 2019
MWF 1pm – 1.50pm, Davis 217

The Basics

Instructor: George Melvin
Contact: gwmelvin@colby.edu
Office: Davis Science Center 212
Office hours: M 10-11am, F 2-3pm or by appointment.
Alternatively, you are welcome to chat with me when my office door is open and I am available.
Course website: http://personal.colby.edu/~gwmelvin
Announcements and handouts can be found at the course website. Please check frequently.
Important dates:
3/11: Diagnostic Test
3/22: Drop Deadline
4/3: Midterm

The Course

Course description: This course is an introduction to the representation theory of finite groups. We will learn the fundamental concepts and theorems - Maschke's Theorem, Schur's Lemma, the Orthogonality Relations, the Dimension Theorem - and gain familiarity with the theory of characters of finite groups. Our approach will utilise Fourier analysis on finite groups: we will learn what this means and why this is an analog of the traditional theory of Fourier series. We will work towards the Young tableaux realisation of the representation theory of the symmetric group S_n and construct all irreducible representations of S_n. In the final weeks of the course we will investigate one (or more) of the following additional topics: representation theory of quivers, representation theory of compact Lie groups, representation theory of Lie algebras, or anything else of interest to students.
Mathematical topics: Review of linear algebra. Definition of (sub)representation, (ir)reducible representations, complete reducibility, indecomposable representations; unitary representations, Maschke's Theorem; morphisms, Schur's Lemma, isomorphic representations; the group algebra, class functions, characters; the regular representation; orthogonality relations of characters and their consequences; character tables. Fourier analysis on finite groups, convolution. Case study: character groups of finite abelian groups. Algebraic integers, the Dimension Theorem. Group actions, permutation representations. The Young tableaux package; (poly)tabloids, Sprecht representations; construction of all irreducible representations of the symmetric group. Introduction to the character theory of the symmetric group and combinatorics. Additional topics TBD.
Prerequisites: Linear algebra and a first course in group theory e.g. MA253, MA333. Let me know if you have questions regarding your preparedness for this course.
Required Course Textbook: *Representation Theory of Finite Groups*, by Benjamin Steinberg. Can be accessed at *Springerlink* (link.springer.com) via Colby portal.

The Skinny

Grading: Your overall grade is given by the following prescription:

Midterm	25%
Project & Talk	25%
Final	25%
Homework	15%
Participation	10%

Examinations: There will be two examinations for this class: a Diagnostic Test and a Midterm. Examinations will take place at the following space/time locations:

What?	When?	Where?
Diagnostic Test	Monday, March 11th, 1-1.50pm	Davis 217
Midterm	Wednesday, April 3rd, 7-9pm	TBA

Examination policy: You must take each Examination at the prescribed location/time. Known scheduling conflicts must be announced to me as soon as possible and additional arrangements will be made on a case-by-case basis. Failure to attend an Examination without excuse results in a failure for the Examination; there will be no make-up Examinations. Exceptions to this policy will only be granted in compelling circumstances.

Diagnostic Test: There will be an in-class, open-book Diagnostic Test on March 11th. The Diagnostic Test will not be counted towards your overall grade and its purpose is to provide you with initial feedback in a test environment and to give me a sense of how everyone is doing: think of the test as Midterm warm-up. The Diagnostic Test will consist of 1-3 proof-writing exercises and 1-3 computational exercises. The Diagnostic Test will be graded; students who perform below their expectations should arrange to meet with me.

Midterm: There will be a two-hour, closed-book Midterm on April 3rd (the week following Spring Break). The Midterm will consist of several proof-writing exercises and several computational exercises. A practice exam will be provided to you before Spring Break to give you a sense of what to expect.

Project & Talk: The course catalogue description of MA434 states that '*Improving one's written and oral communication of mathematics is an integral part of the course.*' As such, you will have the opportunity to work on a project related to course material. A list of project proposals will be made available before Spring Break.

You will be required to submit a written assignment outlining your understanding of your project topic: think of this assignment as a mathematical essay. Solutions to writing assignment problems are expected to be correct, well-written, and presented in good mathematical prose. Your grade will depend on the correctness of your solutions and the quality of your writing. Your writing should follow a coherent logical structure which makes use of complete sentences and follows standard rules of grammar. You will be able to turn-in one draft for feedback and a chance to edit and revise before the assignment's official deadline. Finally, more is required for a perfect grade beyond merely a correct answer.

In addition, you will give a 10-15 minute talk on your project. This talk is expected to be more in the style of a *TED Talk* than a traditional mathematical lecture: the emphasis is on ideas rather than details. You should aim your talk at a more general audience of mathematicians (who may not be algebraists!). Learning how to communicate technical information to a non-technical audience is an extremely important and valuable skill.

After the beginning of the semester, you will be given a specific guidelines concerning the Project & Talk, including grading policies and a description of expectations. Tentatively, the written assignment is due (no earlier than) April 15th and talks will be scheduled for the following week.

Final: There will be a take-home final paper to be submitted during Final Examination week (5/15-20). Further details, including the precise date for submission, will be announced just after Spring Break.

Homework: The goal of homework in this course is to provide you with an opportunity to practice effective communication of abstract mathematical ideas and to gain practice with computations. It is crucial that you take homework very seriously.

Homework will generally be due for submission in class by 1.10pm every Wednesday, unless otherwise indicated, beginning Wednesday, February 13th. **Late homework will not be accepted** and exceptions to this policy will only be granted in exceptional and compelling circumstances. You are **strongly** encouraged to work with your peers on homework. You are warmly encouraged to discuss problems with me. **I expect you to spend at least 6-8 hours on each homework submission. Get started early!**

Suggested collaboration policy:

- You are strongly encouraged to collaborate with your peers. However, you should write up solutions *on your own* and *in your own words*.
- Any collaboration undertaken to obtain a solution on submitted work must be explicitly acknowledged; such acknowledgements will not be penalised. For example, write '*I worked with E. Noether, Archimedes, C. F. Gauss*' on your submission.
- Online forums (eg `math.stackexchange.com`) must not be utilised to complete or check your solutions to submitted work. Consulting Solutions Manuals is **verboten**. Violation of this policy is considered *cheating* in this class and will result in College Academic Violations being issued.

Homework policy: Homework assignments must be written out correctly and your work should follow a coherent logical structure. **Write in complete English sentences** whenever possible. Do not submit solutions containing incoherent and/or unstructured calculations: be kind to your grader (i.e. me).

- All write-ups are to be submitted in hard copy and completed using LaTeX. LaTeX templates will be provided for homework submission. You are allowed to consult online forums (e.g. `stackexchange.com`) for help with LaTeX syntax. You are also allowed to consult with me for any typesetting issues.
- Each paper you turn in should have your name, your statement of collaboration and the due date printed clearly at the top.
- **Your write-ups should be stapled.** It is your responsibility to make sure that your homework is complete and all pages are accounted for.
- If you used technology (e.g. Mathematica, numpy library) in the solution of the problem, please provide the complete printout showing all code, inputs and outputs in sequence, and annotate the printout with comments explaining what is being done in each step.

Attendance: Timely attendance of lectures is mandatory. If you expect regular class attendance to be a problem then let me know immediately. In particular, this applies to athletics-explained absences. Persistent lateness/absence (i.e. ≥ 4 classes) will result in grade penalties, academic warnings and eventual dismissal from the class. Changes to the attendance policy will only be granted in exceptional and compelling circumstances.

No. of Classes Missed	Grade Penalty	Action
4-5	1/3 of letter grade e.g. B becomes B-	Academic Warning 1
6	2/3 of letter grade e.g. B becomes C+	Academic Warning 2
≥ 7	Fail	Dismissal

The Important Stuff

Classroom etiquette: You are expected to be seated at your desk and ready to engage by the beginning of the class. You are expected to be courteous to your classmates and to help foster an inclusive, safe learning environment. Do not talk over each other; do not disregard someone's viewpoint; avoid the use of phrases like *'that's easy'*. Our learning environment will not be competitive.

Mobile and listening devices must be kept silent and non-vibrating throughout the duration of class. Use of your mobile device in-class is not permitted *unless otherwise specified*; if you are expecting a call then let me know. If you wish to use a laptop to take notes and/or document the class then let me know and we can discuss. You must only use your computer as a note-taking device *unless otherwise specified*. Laptops and listening devices should be silent throughout the class. Repeated violation of this policy will reflect negatively on your overall grade.

Learning etiquette: Be honest in your approach to learning. *Ask questions:* whenever you struggle with a topic, if an explanation is unclear, if notation is not defined; you will not be the only one with that question. *Direct questions:* to yourself, to your classmates, to me.

Ask questions even if you think they are stupid.

One of the most difficult tasks to undertake when learning is asking a *useful* question. You can help yourself by adopting the following 3RA strategy when asking questions:

- Reflect: Ask yourself the question: does the question answer itself?
- Write: Write down your question down as a coherent sentence: does this help you see your way to a solution?
- Reformulate: Having constructed a coherent formulation of your question: can you ask a more pointed question?
- Ask: Ask your question!

Get to know your classmates and form study groups: discuss the material, work through problems, ask questions, help each other. If you understand something then challenge yourself by trying to explain your understanding to your peers. If someone struggles with your explanation then reformulate your argument: use examples, visual aids, simple analogies. The onus lies more greatly on the teacher to provide an adequate explanation than it does on the student to comprehend that explanation. If you are seeking explanation then *formulate your question as a coherent sentence.*

"What I cannot create, I do not understand"

Richard Feynman, 1918-1988

Renowned physicist, educator and Nobel Laureate.

Anti-discrimination commitment: I am firmly committed to diversity and equality in all areas of campus life. In this class we will promote an anti-discriminatory environment where everyone feels safe and welcome. I recognize that discrimination can be direct or indirect and take place at both institutional and personal levels. I believe that such discrimination is unacceptable and I am committed to providing equality of opportunity for all by eliminating any and all discrimination, harassment, bullying, or victimization. The success of this policy relies on the support and understanding of everyone in this class. We all have a responsibility not to be offensive to each other, or to participate in, or condone harassment or discrimination of any kind.

Accommodations: Students in this class who have Letters of Accommodation from the Dean of Studies Office are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation *please meet with me within two weeks of the start of the semester* so that we can work together to make arrangements for you. Kate McLaughlin, Associate Director of Access and Disability Services (`kmclaugh@colby.edu`) is the primary contact for accommodations and questions related to educational testing and documentation.

Academic Integrity: Honesty, integrity, and personal responsibility are cornerstones of a Colby education and provide the foundation for scholarly inquiry, intellectual discourse, and an open and welcoming campus community. These values are articulated in the Colby Affirmation and are central to this course. You are expected to demonstrate academic honesty in all aspects of this course. If you are clear about course expectations, give credit to those whose work you rely on, and submit your best work, you are highly unlikely to commit an act of academic dishonesty.

Academic dishonesty includes, but is not limited to: violating clearly stated rules for taking an exam or completing homework; plagiarism (including material from sources without a citation and quotation marks around any borrowed words); claiming another's work or a modification of anothers' work as one's own; buying or attempting to buy papers or projects for a course; fabricating information or citations; knowingly assisting others in acts of academic dishonesty; misrepresentations to faculty within the context of a course; and submitting the same work, including an essay that you wrote, in more than one course without the permission of the instructors.

Academic dishonesty is a serious offense against the College. Sanctions for academic dishonesty are assigned by an academic review board and may include failure on the assignment, failure in the course, or suspension or expulsion from the College.

For more on recognizing and avoiding plagiarism, see the library guide:

http://libguides.colby.edu/avoidingplagiarism

> The Colby Affirmation
>
> Colby College is a community dedicated to learning and committed to the growth and well-being of all its members.
> As a community devoted to intellectual growth, we value academic integrity. We agree to take ownership of our academic work, to submit only work that is our own, to fully acknowledge the research and ideas of others in our work, and to abide by the instructions and regulations governing academic work established by the faculty.
> As a community built on respect for ourselves, each other, and our physical environment, we recognize the diversity of people that have gathered here and that genuine inclusivity requires active, honest, and compassionate engagement with one another. We agree to respect each other, to honor community expectations, and to comply with college policies.
> As a member of this community, I pledge to hold myself and others accountable to these values.

Finally, a parting thought: Remember that MATHEMATICS IS DIFFICULT! We will strive towards a high level of rigor and understanding, and it can be a struggle to wade through the mathematical marsh of complex concepts, technical tricks, and difficult definitions. However, if you are dedicated to your work, exercise your problem-solving abilities frequently, and talk about mathematics with your peers then I guarantee that you will be able to achieve your goals for this course.