Colby

Representation Theory
Fall 2019

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Final Paper

Instructions:

1. Submit a 5-8 page paper summarizing a particular topic and outlining the proof of a previously unknown result.
2. Your paper should be single-spaced, written in LaTeX, and contain appropriate references.
3. You are allowed to consult any materials that you wish (e.g. textbooks, online), but all resources used should be properly cited. You are allowed to collaborate on your projects but you should cite who you work with whenever appropriate. You are very welcome to ask me (or anyone else) for guidance.
4. Submit your paper by Monday 5/20, 4pm.

Topics:

1. *Fourier analysis for finite non-abelian groups:*

   First: summarize Fourier analysis for finite abelian groups (e.g. the main results); **Second:** explain, in your own words, the proof of Theorem 5.5.6 in the course textbook. This theorem states that the Fourier transform for finite non-abelian groups defines an isomorphism

   $$\mathbb{C}[G] \xrightarrow{\sim} \prod_{i=1}^{m} M_{d_i}(\mathbb{C})$$

   Reference: *Representation Theory of Finite Groups* - B. Steinberg, Springer

2. *Cayley graphs & circulant matrices:*

   Explain, in your own words, the proofs of Theorem 5.4.10 and Corollary 5.4.11 in the course textbook. These results provide a connection between representation theory and graph theory.

   Reference: *Representation Theory of Finite Groups* - B. Steinberg, Springer

3. *Induced representations:*

   Explain, in your own words, Sections 8.1 and 8.2 of the course textbook. You should include a proof of Frobenius Reciprocity (Theorem 8.1.3), the definition of the induced representation (middle of page 101), and either Example 8.2.3 or Example 8.2.4.

   If you're eager, you can include the proof of Theorem 8.2.5 for additional **awesome points** (not actually worth anything tangible, unfortunately).

   Reference: *Representation Theory of Finite Groups* - B. Steinberg, Springer