Colby

MA121: Spring 2019

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Practice Midterm 2

Disclaimer: This Practice Midterm consists of problems of a similar difficulty as will be on the actual midterm. However, **problems on the actual midterm may or may not be quite different in nature, and may or may not focus on different course material**. However, the actual midterm will have a similar format: one true/false problem, one short-answer problem, three long-answer problems.

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1. True/False (no justification required)
   (a) The function $f(x) = |x + 1|$ is not differentiable.
   (b) Let $f(x) = e^x + 5$, where $-1 \leq x \leq 10$. Then, $x = 10$ is a local maximum of $f(x)$.
   (c) Let $f(x)$ be differentiable everywhere. Suppose $f(0) = f(5) = 5$. Then, there exists $0 < u < 5$ such that $f'(u) = 5$.
   (d) If $f''(p) > 0$ then p is a local minimum.
   (e) Let $y = \sin(\cos(x))$. Then, $\frac{dy}{dx} = \sin(\cos(x)) + \cos(\sin(x))$.

2. (a) Let $g(t) = e^{\cos(t^3)}$. Determine $g'(t)$.
   (b) Let $f(x) = \frac{\sin(x^2)}{e^x}$. Determine $f'(x)$.

3. A piece of wire having length 10m is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.
   (a) Let x be the side length of the square. Show that the total area A of the square and triangle can be expressed as
   $$A = x^2 + \frac{\sqrt{3}}{4}\left(\frac{10 - 4x}{3}\right)^2$$
   Hint: the area of an equilateral triangle having side length a is $\frac{\sqrt{3}}{4}a^2$.
   (b) Determine the side length of the square giving the largest possible area A.

4. Let $g(x) = x\sqrt{4 - x^2}$, $-1 \leq x \leq 2$. Find global maximum and global minimum for $g(x)$.

5. Let $f(x) = xe^{-x^2}$.
   (a) Determine the local maxima/minima of $f(x)$.
   (b) Determine the inflection points of $f(x)$.
   (c) Using what you've found, sketch the graph of $f(x)$. (*You may find the following information useful:* $\lim_{x \to \pm\infty} f(x) = 0$.)