Colby

MA121: Spring 2019

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Practice Midterm 1

Disclaimer: This Practice Midterm consists of problems of a similar difficulty as will be on the actual midterm. However, **problems on the actual midterm may or may not be quite different in nature, and may or may not focus on different course material**. However, the actual midterm will have a similar format: one true/false problem, one short-answer problem, three long-answer problems.

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1. True/False (no justification required)

   (a) If $\lim_{x\to a} f(x)$ exists then $f'(a)$ exists.

   (b) If $\log_2(x) = 5$ then $x = 32$.

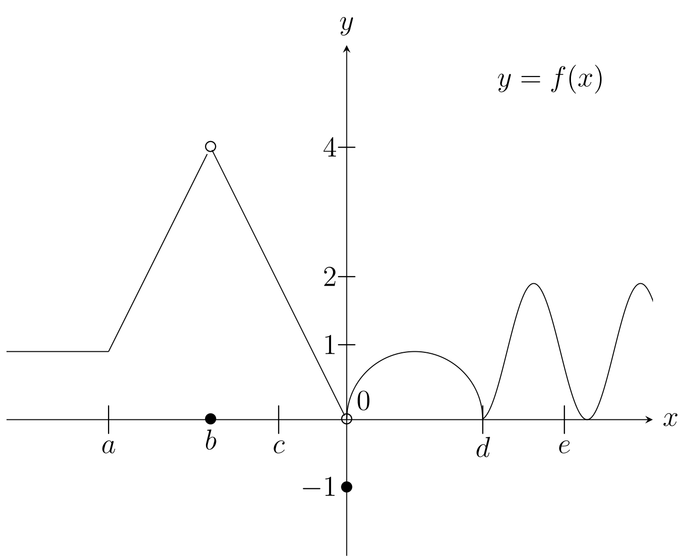
   (c) If $f(5) = -1$ and $f(x)$ is continuous at $x = 5$ then $\lim_{x\to 5} f(x) = -1$.

   (d) $\frac{d}{dx}x^e = x^e$.

   (e) $\frac{d}{dx}(5xe^x + \pi^e \cos(x)) = 5e^x - e\pi^{e-1}\sin(x)$

2. The graph of a function $f(x)$, defined for all x, is shown below. Using the given information, circle the true statements:

   - $\lim\limits_{x\to 0} f(x) = f(d)$
   - $\lim\limits_{x\to b^+} f(x) > f(0)$
   - $\lim\limits_{x\to a} f(x) = f(a)$
   - $\lim\limits_{x\to \infty} f(x) = 0$
   - $\lim\limits_{x\to d} f(x) = f(d)$
   - $f'(a)$ DNE
   - $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to -\infty} f'(a-1)$
   - $\lim\limits_{h\to 0} \dfrac{f(e+h) - f(e)}{h} < \lim\limits_{x\to c} f(x)$
   - $\lim\limits_{x\to b} f(x) = 4$



3. Let k, l be constants.

(a) Using limit laws, or otherwise, determine

$$\lim_{x \to 1} \frac{x^2 + kx - 2}{x^2 + 5x - 5}$$

Note: your answer will be in terms of k.

(b) Define

$$f(x) = \begin{cases} \frac{x^2+kx-2}{x^2+5x-5}, & x \neq 1 \\ l, & x = 1 \end{cases}$$

Determine all values of k, l so that $f(x)$ is continuous at $x = 1$. Be careful to justify why you know that $f(x)$ will continuous at $x = 1$ for these values of k, l.

4. Let $f(x) = \sqrt{2 - x}$.

   (a) What is the domain of $f(x)$?

   (b) Determine the tangent line to the graph $y = f(x)$ at $(1, f(1))$.

   (c) Determine a formula for the inverse function $f^{-1}(x)$ and specify its domain.

5. Let $f(x) = \begin{cases} -x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$

   (a) Using the limit definition of the derivative, explain why $f'(0)$ does not exist.

   (b) Draw the graph of the derivative $f'(x)$.