Colby

MA121B: Spring 2019
Homework 8

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Some thoughts and advice:

- You should expect to spend at least several hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You must write your solutions *on your own* and *in your own words*.
- If you would like more practice then let me know.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

Submit solutions to the following problems on **Friday, April 26th**.

1. Problems 1, 2, 3, 4, 14, 15, 19, 36, 37, 40, 43, 48, 49, 51, 52-55 in Section 5.2
2. In this problem you will show directly (i.e. using right-hand sums) that

$$\int_0^1 x^2 dx = \frac{1}{3}$$

   You will need the following Magic Formula:

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

   (a) Subdivide the interval $0 \le x \le 1$ into 10 subintervals having equal length. Label the endpoints of the subintervals $x_0, \ldots, x_{10}$.

   i. Explain why $x_i = \frac{i}{10}$, for $i = 0, 1, 2, \ldots, 10$.

   ii. Show that $R_{10} = \dfrac{1}{10^3} \displaystyle\sum_{i=1}^{10} i^2$

   iii. Use the Magic Formula to compute R_{10}

   (b) Let n be a positive integer. Subdivide the interval $0 \le x \le 1$ into n subintervals having equal length. Label the endpoints of the subintervals $x_0, \ldots, x_n$.

   i. Show that $x_i = \frac{i}{n}$, for $i = 0, 1, 2, \ldots, n$.

   ii. Show that $R_n = \dfrac{1}{n^3} \displaystyle\sum_{i=1}^{n} i^2$

   iii. Use the Magic Formula to show that

$$R_n = \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

   iv. By considering $\lim_{n\to\infty} R_n$, explain why $\displaystyle\int_0^1 x^2 dx = \frac{1}{3}$.