Math 121B: Single-Variable Calculus
Spring 2019

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Supplementary References:
- *Calculus*, Hughes-Hallet et al, Section 1.7, 1.8

Keywords: *continuous at a point, continuous, Intermediate Value Theorem*

Properties, applications of continuous functions

George-given Truths:
• all 'nice' functions - e.g. polynomial, rational, exponential, trigonometric, logarithmic - are continuous everywhere in their domain.
• piecewise functions are continuous, except possibly at jump discontinuities.

• Let $f(x), g(x)$ be continuous at $x = c$. Then,

- $f(x) \pm g(x)$ is continuous at $x = c$.
- $f(x)g(x)$ is continuous at $x = c$.
- $\frac{f(x)}{g(x)}$ is continuous at $x = c$, provided $g(c) \neq 0$.
- $kf(x)$ is continuous at $x = c$, for k constant.

• Let $f(x)$ be continuous at $x = c$, $g(x)$ continuous at $x = f(c)$. Then, $g(f(x))$ continuous at $x = c$.

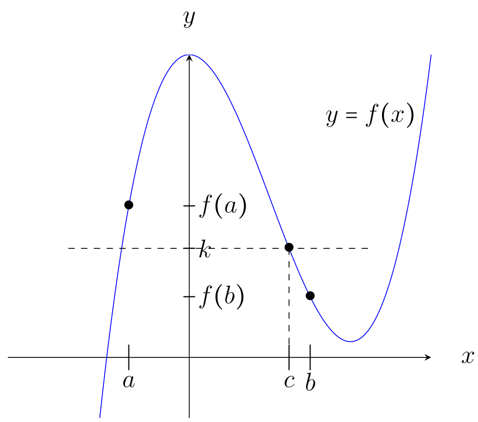
Example:

1. Determine
$$\lim_{x\to 1} \frac{x^7 - 9x + 5}{3x^2 + x}$$
Solution: Let $f(x) = \dfrac{x^7 - 9x + 5}{3x^2 + x}$. Then, $f(x)$ is defined at $x = 1$ and continuous at $x = 1$ since it's a rational function. Hence
$$\lim_{x\to 1} f(x) = f(1) \qquad \text{(i.e. (C2) holds for } f(x) \text{ at } x = 1)$$
$$\implies \lim_{x\to 1} \frac{x^7 - 9x + 5}{3x^2 + x} = f(1) = \frac{1^7 - 9\cdot 1 + 5}{3\cdot 1^2 + 1} = \frac{-3}{4}$$

2. Determine
$$\lim_{x\to -2} x2^x$$
Solution: Let $f(x) = x2^x$. Then, $f(x)$ is defined at $x = 1$ and continuous at $x = 1$ since it's the product of $g(x) = x$ and $h(x) = 2^x$. Hence
$$\lim_{x\to -2} f(x) = f(-2) \qquad \text{(i.e. (C2) holds for } f(x) \text{ at } x = -2)$$
$$\implies \lim_{x\to -2} x2^x = f(-2) = (-2)\cdot 2^{-2} = -\frac{1}{2}$$

Intermediate Value Theorem (IVT):

• *Let $f(x)$ be a continuous function defined on the domain $a \leq x \leq b$. Suppose that $f(a) < z < (b)$ or $f(a) > z > f(b)$. Then, there is $a < c < b$ satisfying $f(c) = z$.*



Application: Let $f(x) = x^5 + 7x + 6$. Then, $f(x)$ continuous everywhere. Observe:

$$f(-1) = -2 \qquad \text{and} \qquad f(0) = 6$$

Let $z = 0$, so that $f(-1) < z < f(0)$. Therefore, by IVT, there is $-1 < c < 0$ so that $f(c) = 0$ i.e. the equation $f(x) = 0$ has a solution in interval $-1 < x < 0$.