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February 20 Summary

Supplementary References:

- *Calculus*, Hughes-Hallet et al, Section 1.7, 1.8

Keywords: *continuous at a point, continuous*

Continuity

• Let $f(x)$ be a function, $x = c$ in the domain of $f(x)$. Say that $f(x)$ **is continuous at** $x = c$ if

(A) $\lim_{x\to c} f(x) = L$ exists, and

(B) $L = f(c)$.

If $f(x)$ is continuous for every c in its domain then we say $f(x)$ is **continuous**.

Example:

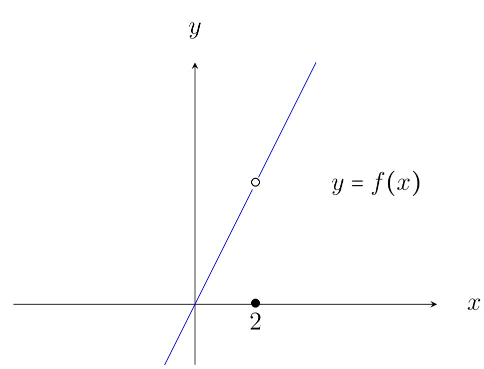
1. $f(x) = 2x$, domain $= \mathbb{R}$. Our computation from a previous lecture showed that $\lim_{x\to 2} f(x) = 4$. Since $f(2) = 4$, we have that $f(x)$ is continuous at $x = 2$.

   More generally: using Limit Laws we can show that, for any real number c,

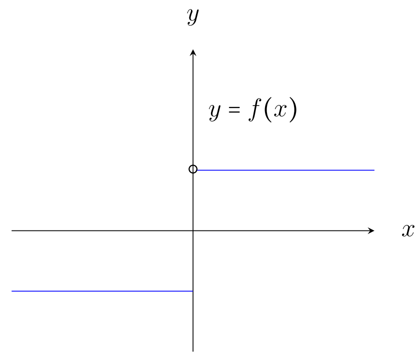
   $$\lim_{x\to c} 2x \overset{LL1}{=} 2\lim_{x\to c} x \overset{LL6}{=} 2c$$

   Thus, since $f(c) = 2c$, $f(x) = 2x$ is continuous at every point in its domain. Hence, $f(x)$ is continuous.

2. Let $f(x) = \begin{cases} 2x, & x \neq 2 \\ 0, & x = 2 \end{cases}$. Then, $\lim_{x\to 2} f(x) = 4$. However, $f(2) = 0 \neq 4$. Hence, $f(x)$ is not continuous at $x = 2$.



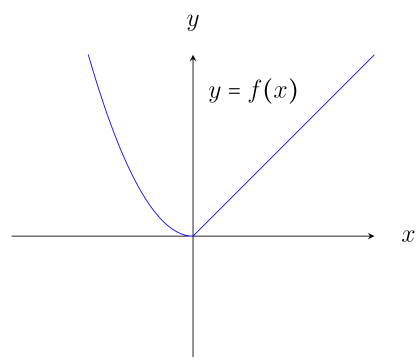
3. Consider $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x \leq 0 \end{cases}$. In this case, $\lim_{x\to 0^+} f(x) = 1$ while $\lim_{x\to 0^-} f(x) = -1$, which we can determine using Limit Laws. Therefore, $\lim_{x\to 0} f(x)$ does not exist and $f(x)$ is not continuous at $x = 0$.



4. Consider $f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$.

   - $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 = \left(\lim_{x \to 0^-} x\right)^2 \overset{LL6}{=} 0^2 = 0$
   - $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x \overset{LL6}{=} 0$

   Hence, both one-sided limits exist and are equal, and $\lim_{x \to 0} f(x) = 0$. Since $f(0) = 0$, $f(x)$ is continuous at $x = 0$.



George-given Truths:

• all 'nice' functions - e.g. polynomial, rational, exponential, trigonometric, logarithmic - are continuous everywhere in their domain.

• piecewise functions are continuous, except possibly at jump discontinuities.