Math 121B: Single-Variable Calculus
Spring 2019

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Supplementary References:
- *Calculus*, Hughes-Hallet et al, Section 1.8

Keywords: *limits at infinity, rigorous definition of limit*

Limits contd.

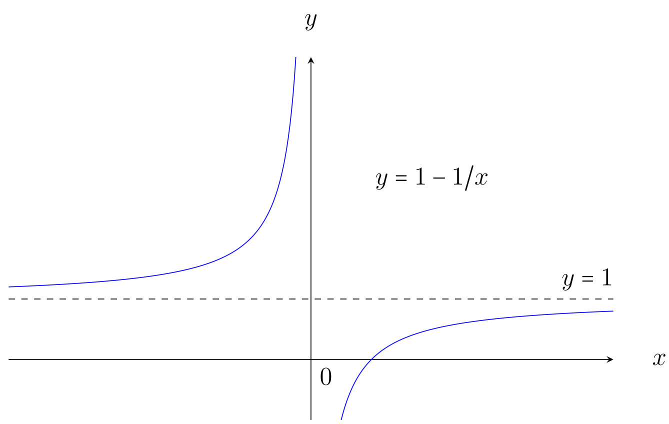
Other types of limits: limits at infinity

• If $f(x)$ approaches a finite real number L as x gets very large then we write $\lim_{x\to\infty} f(x) = L$.

• If $f(x)$ approaches a finite real number L for $x < 0$ and as $|x|$ gets very large then we write $\lim_{x\to-\infty} f(x) = L$.

Example: Let $f(x) = 1 - \frac{1}{x}$, $x \neq 0$. As x gets very large, $\frac{1}{x}$ becomes very small and $1 - \frac{1}{x}$ gets close to $1 - 0 = 1$. Hence, $\lim_{x\to\infty} f(x) = 1$.

Similarly, as x gets very large in the negative direction, the quantity $\frac{1}{x}$ is negative and gets closer and closer to 0. Hence, $1 - \frac{1}{x}$ gets close to $1 + 0 = 1 \implies \lim_{x\to-\infty} f(x) = 1$ also.



Example: How to compute limits without the graph? We have some **rules of the road** or **allowed moves** known as **Limit Laws** (see handout).

For example, suppose we want to compute

$$\lim_{x\to 1} \frac{2x^3 - 5x + 7}{6x^2 + 3}$$

Use Limit Laws:

- numerator:

$$\begin{aligned}
&\lim_{x\to1}(2x^3-5x+7)\\
&=\lim_{x\to1}2x^3-\lim_{x\to1}5x+\lim_{x\to1}7, \quad \text{by LL2}\\
&=2\lim_{x\to1}x^3-5\lim_{x\to1}x+7, \quad \text{by LL1, LL5}\\
&=2(\lim_{x\to1}x)^3-5\cdot1+7, \quad \text{by LL3, LL6}\\
&=2\cdot1^3-5+7, \quad \text{by LL6}\\
&=4
\end{aligned}$$

- denominator:

$$\begin{aligned}
&\lim_{x\to1}(6x^2+3)\\
&=\lim_{x\to1}6x^2+\lim_{x\to1}3, \quad \text{by LL2}\\
&=6\lim_{x\to1}x^2+3, \quad \text{by LL1, LL5}\\
&=6(\lim_{x\to1}x)^2+3, \quad \text{by LL3}\\
&=6\cdot1^2+3, \quad \text{by LL6}\\
&=9\neq0
\end{aligned}$$

- combine: hence, since $\lim_{x\to1}(6x^2+3)\neq0$, we use LL4 to get

$$\lim_{x\to1}\frac{2x^3-5x+7}{6x^2+3}=\frac{\lim_{x\to1}(2x^3-5x+7)}{\lim_{x\to1}(6x^2+3)}=\frac{4}{9}$$

• **Remark:** the fact that this limit is equal to what we'd obtain by inputting $x=1$ into the expression and evaluating is a consequence of the fact that $f(x)=\frac{2x^3-5x+7}{6x^2+3}$ is **continuous at** $x=1$.

Continuity

Let $f(x)$ be a function, $x=c$ in the domain of $f(x)$. Say that $f(x)$ **is continuous at** $x=c$ if

(C1) $\lim_{x\to c}f(x)=L$ exists, and

(C2) $L=f(c)$

If $f(x)$ is continuous for every c in its domain, we say that $f(x)$ is **continuous**.