As promised, I have written an itemized list of topics we've covered in Math 381 since the previous midterm exam. The exam will cover the entirety of Sections 4.2 through 5.2 of the course notes. This list of topics (and the proportion of time we've spend on them since the previous midterm exam) will align with the problems you will see on the second midterm exam. In studying for the exam, please note that I consider the homework exercises and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

Theory

1. You should know all definitions (precisely). In particular, you should know the definitions of the following terms and be able to list several examples/applications for each:

   a. What it means for two events to be dependent (You should also understand this conceptually, i.e., think of examples).

   b. Given a finite collection $A_1, A_2, \dots, A_n$ of events, know what it means for this collection to be independent.

   c. The meaning of independent trials. That is, the process by which you perform the same experiment over and over again independently. As we discussed in class, there are ways to encode the sample space as a Cartesian product (i.e., with ordered n-tuples).

   d. You should know the definition of random variable (on a sample space). Note that – the definition of random variable does not involve a probability measure – only the sample space.

   e. Range of a random variable.

   f. How to interpret and write events in terms of random variables, i.e., for any k,

   $$\{X = k\} = \{\omega \in \Omega : X(\omega) = k\}$$

   g. Given a random variable X on a countable sample space Ω which is equipped with probability measure $\mathbb{P}$, you should know the following definition of expectation:

   $$\mathbb{E}(X) = \sum_{\omega \in \Omega} X(\omega)\mathbb{P}(\{\omega\})$$

   which is defined (and exists as a real number) whenever

   $$\sum_{\omega \in \Omega} |X(\omega)|\mathbb{P}(\omega) < \infty.$$

   More generally, for any reasonable function of the random variable $\varphi(X)$ where $\varphi : \mathbb{R} \to \mathbb{R}$, you should know the definition of $\mathbb{E}(\varphi(X))$.

   h. You should know the definition of the variance of X, $\mathrm{Var}(X)$, and the moments of X, i.e., the numbers $\mathbb{E}(X^n)$ for each $n = 0, 1, 2, \dots$.

   i. For a random variable X on a countable sample space Ω with probability $\mathbb{P}$, should know the definition of X's probability mass function $p_X(k)$.

   j. Know what it means for a random variable X to be Bernouilli with parameter p, i.e., $X \sim \mathrm{Ber}(p)$.

   k. Know what it means for a random variable X to be Binomial with parameters n and p, i.e., $X \sim \mathrm{Bi}(n, p)$.

   l. Know what it means for a random variable X to be Geometric with parameter p, i.e., $X \sim \mathrm{Geo}(p)$.

   m. Know what it means for a random variable X to be Poisson with parameter λ, i.e., $X \sim \mathrm{Pois}(\lambda)$.

   For the four preceding items, you should also be very familiar with what each random variable measures. In other words, you should have a solid idea of the types of experiments that lead to these random variables and what the random variables quantify for these experiments. Note: We will derive the Poisson random variable next week.

2. We have discussed many properties and results in this course (concerning the concepts listed above). For Wednesday's exam, you should know, in particular, the following results (properties derived, theorems, propositions, etc.). Note: I have indicated (with an asterisk "$*$") which properties you should be able to show/argue in detail. This, in general, does not mean prove but you should have a VERY solid idea of how things are gotten and which properties are used and when.

   * Proposition 4.13.
   - Theorem 4.14
   * Proposition 5.8
   * Proposition 5.11
   - Proposition 5.12
   * Theorem 5.14
   * Corollary 5.15
   * Corollary 5.16
   * Proposition 5.19
   - Theorem 5.20
   * Proposition 5.23
   * Proposition 5.25
   * Proposition 5.27

Computations and Problem Solving

3. The following are computations that you should know how to do (accurately and quickly).

   (a) Given an experiment and a collection of events (collection not consisting of more than three events), you should be able to determine if the collection is independent.

   (b) You should be able to make arguments and cook up independent random variables based on independent (or simply Bernoulli) trials.

   (c) Given a random variable X on a countable sample space Ω which is equipped with a probability measure $\mathbb{P}$, you should be able to compute probabilities associated with the random variable, i.e., for any subset I of real numbers, you should be able to compute $\mathbb{P}(X \in I)$.

   (d) Given a random variable X on a countable sample space Ω equipped with a probability measure $\mathbb{P}$, you should be able to compute the cumulative distribution function of the random variable.

   (e) Given a random variable X on a countable sample space Ω equipped with a probability measure $\mathbb{P}$, you should be able to compute the mean $\mathbb{E}(X)$, the moments $\mathbb{E}(X^n)$, the variance $\mathrm{Var}(X)$, and other expectations of the form $\mathbb{E}(\varphi(X))$ for functions $\varphi : \mathbb{R} \to \mathbb{R}$.

   (f) Given a discrete random variable X, you should know how to show/check that its probability mass function satisfies the normalization condition, $\sum_x p_X(x) = 1$. In particular, this means you should know how to work with certain finite sums using, say, the binomial theorem and infinite sums using some basic results of power series.

   (g) You should be able to compute the mean and variance (directly) for all of the major discrete random variables we've encountered. We have usually argued using what I've called "the derivative trick", though there are several ways to do each. Whatever your method, you should be very comfortable with those computations.