As promised, I have written an itemized list of topics we've covered in Math 381 since the beginning of the semester. The exam will cover the entirety of Chapters 1, 2, 3 and Section 4.1 of Chapter 4 in the course notes. This list of topics (and the proportion of time we've spend on them since the beginning of the semester) will align with the problems you will see on Wednesday's midterm exam. In studying for Wednesday's midterm exam, please note that I consider the homework exercises and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

Theory

1. You should know all definitions (precisely). In particular, you should know the definitions of the following terms and be able to list several examples/applications for each:

   a. Sample space, outcome, event.

   b. Basic definitions/operations on events (subset/superset, intersections, unions, complements, set differences).

   c. What it means for a collection of events to be pairwise disjoint.

   d. You should know what a partition of a sample space Ω is. This is, by definition, a collection $S_1, S_2, \dots$ of events (either finite or infinite) which are pairwise disjoint, i.e., $S_k \cap S_j = \varnothing$ for each $k \neq j$, and whose union is all of Ω, i.e.,
   $$\Omega = \bigcup_j S_j.$$

   e. You should know the definition of the indicator function for an event E.

   f. You should know what "finite", "countable", "countably infinite", and "uncountable" means. You should worry about these things too much, but you should have some good ideas about the definitions/concepts.

   g. Probability measure on a sample space. Though I did present two definitions (the first being a "first attempt"), for the midterm you need to know Definition 2.5.

   h. For a finite sample space Ω, know that definition of the (uniform) probability $\mathbb{P}$ on Ω. Remember, this is the probability in which all outcomes are equally likely.

   i. What it means for a countably infinite collection of events $E_1, E_2, \dots$ to be nested increasing or nested decreasing.

   j. You should be very familiar with sampling with and without replacement and with and without order.

   k. The factorial.

   l. Binomial Coefficient, i.e., for integers $0 \le k \le n$, $\binom{n}{k}$.

   m. For an events E and F with $\mathbb{P}(F) > 0$, you need to know the definition of the conditional probability of E given F, $\mathbb{P}(E|F)$.

2. We have discussed many properties and results in this course (concerning the concepts listed above). For Wednesday's exam, you should know, in particular, the following results (properties derived, theorems, propositions, etc.). Note: I have indicated (with an asterisk "$*$") which properties you should be able to show/argue in detail. This, in general, does not mean prove but you should have a VERY solid idea of how things are gotten and which properties are used and when.

   * Proposition 1.4.
   * Proposition 1.5.
   * Proposition 1.6 (De Morgan's laws).
   - Proposition 1.9.
   * Proposition 2.2.

* Proposition 2.3.
* Proposition 2.4.
* The results in Exercise 2.3 (Monotonicity of Measure and the two-element version of Booles' inequality.
- Lemma 2.8.
- Proposition 2.9.
* Proposition 2.10 (Note, this was the subject of Exercise 2.3).
- Theorem 2.11
* The result about partitions in Exercise 2.5.
* Theorem 2.12.
- Principle 3.1.
* You should be very comfortable with all of the counting arguments in Sections 3.1.1-3.1.4 and the results obtained in those subsections.
- Theorem 3.2.
- Proposition 4.2
* Theorem 4.3.
- Proposition 4.4 (you should know the proof for three events E_1, E_2, E_3).
* Theorem 4.6.
- Theorem 4.8 (this is really just a corollary of Theorem 4.6 and the first version of Bayes').

Computations and Problem Solving

3. The following are computations that you should know how to do (accurately and quickly).

   (a) You should know how to compute probabilities of (lots of) events in equally likely sample spaces.

   (b) Any time an event is described as "at least", you should be comfortable calculating the probability of that event by computing the probability of its complement.

4. You should be familiar and comfortable with most every example we have done in lecture and covered in homework. This list includes but is not limited to:

   (a) Basic counting problems. This includes: How many ways there are to order things (including whether or not there are indistinguishable subcollections). How many ways to choose (where order doesn't matter) a certain number of items (including the case in which some of those items are indistinguishable).

   (b) You should be VERY familiar with the basic events (and their probabilities) in all of our simple coin-flipping and dice-rolling experiments. To this end, you should know and be comfortable with every example I have covered in class and you've done in homework regarding these examples.

   (c) You should know the birthday problem.

   (d) If I described a reasonably simple experiment, you should be able to model the experiment by describing a sample space Ω. You should be able to model the likelihood of outcomes on the sample space by defining a probability measure on Ω.

   (e) Finally, you should do the first four exercises in Chapter 4 in preparation for the exam.