Welcome to Math 311!

This is the zeroth homework assignment for Math 311. It is due by 11:00 AM on September 11th and you will submit your homework assignment to the appropriate box outside my office door (Davis 209), according to the directions in the syllabus. The first exercise functions as a means of introduction. The second exercise is designed to place differential equations in a context which is relevant to you. The third and forth exercises aim to give you a warm up, i.e., to help you dust off your math skills. And the fifth and sixth exercises are taken from the course notes (in Chapter 1) – which we will begin in earnest on Friday. You can expect future assignment to be longer (about 1.5 times as long) and so I encourage you to budget your time for this course appropriately from the start. As always, please attend office hours (and do so early) if you get stuck. More importantly, have fun!

Exercise 1 (Introductions). With the aim of getting to know all of you, please answer the following questions.

1. What is your preferred name, i.e., what should I call you in class? What are your preferred pronouns?
2. What is an interesting fact about you?
3. Why are you taking this course? If your answer is "It's required for my major", what major are you referring to?
4. Beyond mathematics, what subject is most fascinating to you?
5. What was the last math class you took? How did you like it?
6. What role do you think differential equations will play in your life after this class?
7. Is there anything you wish me to know about you as a student of differential equations?[†]
8. Do you have any dog* allergies (or any fear of dogs)?

Exercise 2 (A Writing Exercise). For this first exercise, you will identify an interesting place (for you) in which differential equations show up and are relevant to your studies. Identify a course you are currently taking or have recently taken outside of Math & Stats (e.g., economics, biology, chemistry, physics, geology, computer science) and find a differential equations or a system of differential equations which appears as part of the subject matter[†]. Write a one-page description of this differential equation and the context in which it appears. In your description, make sure to answer the following questions:

1. From what subject was this differential equation taken?
2. What does the differential equation model and why is it interesting?
3. What does the independent variable (or variables) represent?
4. What does a solution (also called a dependent variable) represent?
5. What role do initial conditions (or boundary conditions) play in the model?
6. Does the text attempt to find a closed-form[‡] solution? If so, what is the solution?
7. If the answer to the previous question is "no", does the text discuss dynamical systems analysis or numerical approximations for solutions? What are the take-aways of this discussion?

Please make sure that your discussion is well-written, clear and accessible to someone not well-versed in the subject you chose. If you are stuck finding a differential equation to write about, please visit me during office hours so I can help. As always, make sure to cite any sources you use.

[†]e.g. "I cannot see orange, so don't use that marker," or "I have a dreadful fear of trigonometric functions," "I am a visual learner" "I am on the (insert sport here) team," etc.

*I ask this because sometimes I bring my dog into office hours and, before I bring her in, I want to make sure that everyone is okay with it.

[†]Flip through your textbooks/course notes!

[‡]A solution in terms of elementary functions, e.g., exponentials, logarithms, polynomials, rational functions, trigonometric functions.

Exercise 3 (Practice with logarithms). As the exponential function e^x is ubiquitous in differential equations, its inverse, the natural logarithm[§] $\log x$, will play an essential role for us. It is therefore a necessity that you have a solid understanding of logarithms and their various properties. We shall make use of the following two fundamental (and defining) properties of the natural logarithm in this course:

i. The inverse property:

$$a = e^b \qquad \text{if and only if} \qquad b = \log a$$

for all real numbers b and positive numbers a.

ii. The integral:

$$\log x = \int_1^x \frac{1}{t}\, dt$$

for $x > 0$.

Please do the following:

1. Using the property that $e^{x+y} = e^x e^y$, use the inverse property to show that

$$\log(AB) = \log(A) + \log(B)$$

for any $A, B > 0$.

2. Using the property that $e^{px} = (e^x)^p$, use the inverse property to show that

$$\log(A^p) = p \log(A)$$

for any $A > 0$ and real number p.

3. Using the two previous properties, show that

$$\log(A/B) = \log(A) - \log(B)$$

for $A, B > 0$.

4. What is $\log 1$? What is $\log e$? Please explain using properties above.

5. Please use the properties you've just shown to expand the following:

a. $\log\left(\dfrac{x^4 y^3}{1+y^2}\right)$

b. $\log\left(\dfrac{\sqrt{x}\sqrt[3]{y^2}}{z^4}\right)$

c. $\log\left((xy^2)^{1/3}\right)$

d. $\log\left(x\sqrt{\dfrac{\sqrt{x}}{z}}\right)$

6. Solve the following for x.

a. $\log x + \log(x-1) = \log(4x)$

b. $2\log x = \log 2 + \log(3x-4)$

c. $4 = \log e^{x^2}$

d. $2\log 5 + \dfrac{1}{2}\log 9 - \log 3 = \log x$

[§]In this class, I will almost always write log instead of ln to mean the logarithm of base e (not 10).

Exercise 4 (Integration Practice). Compute the following indefinite integrals. Each integral can be done by hand (and I highly encourage you to do this), but it's okay to use technology to help as long as you include all relevant source code.

a. $$\int x + x^{1/3}\,dx$$

b. $$\int \frac{t}{\sqrt{1+t^2}}\,dt$$

c. $$\int \sin x \cos x\,dx$$

d. $$\int x \sin x\,dx$$

e. $$\int \frac{6x-4}{2+4x-3x^2}\,dx$$

f. $$\int \frac{1}{3-6x}\,dx$$

g. $$\int \frac{\ln x}{x}\,dx$$

h. $$\int \ln x\,dx$$

Exercise 5. Please do Exercise 1 in the Course Notes.

Exercise 6. Please do Exercise 2 in the Course Notes.