As promised, I have written an itemized list of topics we've covered in Math 135 since the beginning of the semester. As we will discuss on Monday, the exam will cover Chapter 1 of Abbott's book, several sections of "Introduction to Mathematics: Number, Space, and Structure" by Scott Taylor (Sections 2.4, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 4.1, 4.2, 4.3, 4.4, 4.5, 9.1, 9.2, and 9.3), and the other handouts appearing on the course website. This list of topics (and the proportion of time we've spend on them since the beginning of the semester) will align with the problems you will see on the midterm exam. In studying for the midterm, please note that I consider the homework exercises and the everything I've covered in lecture to be the best source of practice (problems, proofs, etc). If you know how to approach each problem/exercise/proof, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution/proof, you should perform well on the exam.

Definitions:

The following list enumerates all the definitions you need to know by heart. In particular, you should make sure to know all quantifiers involved in the definitions and the order in which they appear. Also, for each definition, you should be able to come up with several examples satisfying the definition (and hopefully things that don't satisfy the definition).

1. Given an interval $I = [a, b]$, you should know what a partition P of I is.
2. For a bounded real-valued function f on I and a partition P, you should know how to define (and compute)
   (a) The upper Darboux sum $\mathcal{U}(f, P)$.
   (b) The lower Darboux sum $\mathcal{L}(f, P)$.
   (c) For any choice of $t_1, t_2, \ldots, t_n$ with $t_k \in [x_{k-1}, x_k]$, the Riemann sum
   $$\sum_{k=1}^{n} f(t_k)\Delta_k.$$
3. You should know what the upper and lower Riemann/Darboux integrals are. In three different notations, these are (respectively)
   $$\mathcal{U}(f) = \overline{\int_a^b f(x)dx} = (\mathcal{U})\int_a^b f(x)\,dx := \inf_P \mathcal{U}(f, P)$$
   and
   $$\mathcal{L}(f) = \underline{\int_a^b f(x)dx} = (\mathcal{L})\int_a^b f(x)\,dx := \sup_P \mathcal{L}(f, P).$$
4. You should know what it means for a function f to be Riemann/Darboux integrable on an interval $I = [a, b]$. Note that, in class, we've been writing $R(I)$ to denote the set of (real-valued) Riemann/Darboux integrable functions on I and so we would say, for example, that $f \in R(I)$ if and only if f is a bounded real-valued function for which $\mathcal{U}(f) = \mathcal{L}(f)$. In this case, the Riemann/Darboux integral of f on I is the number
   $$\int_a^b f(x)\,dx = \int_a^b f = \mathcal{U}(f) = \mathcal{L}(f).$$
5. In some basic cases, you should know what it means for a function to be improperly Riemann integrable and absolutely Riemann integrable. For example, given a function $f : [0, \infty) \to \mathbb{R}$, we say that f is *improperly Riemann integrable* on $[0, \infty)$ if
   $$\int_0^\infty f(x)\,dx = \lim_{t\to\infty}\int_0^t f(x)\,dx$$
   exists and the value of this limit is called the improper Riemann integral over $[0, \infty)$. If f is replaced by $|f|$, then the existence of the limit
   $$\int_0^\infty |f(x)|\,dx = \lim_{t\to\infty}\int_0^t |f(x)|\,dx$$
   means that f is absolutely Riemann integrable on $[0, \infty)$ and the absolute integral is defined to be this number.

6. You should know what it means for a complex-valued function $f : I \to \mathbb{C}$ to be continuous, differentiable, and integrable. Also, for such a function, you should know how to define (all of this is in Homework 2) the derivative f' and the integral $\int_a^b f$.

7. Given a sequence of complex-valued functions on $E \subseteq \mathbb{R}$, $\{f_n\}$ and another function $f : E \to \mathbb{C}$, you should know

   (a) What it means for $\{f_n\}$ to converge to f pointwise on E.

   (b) What it means for $\{f_n\}$ to converge uniformly to f on E.

   (c) What it means for the sequence $\{f_n\}$ to be uniformly Cauchy on E.

   (d) What it means for the series $\sum_n^\infty f_n(x)$ to converge on E.

   (e) What it means for the series $\sum_n^\infty f_n(x)$ to converge uniformly on E.

   (f) What it means for the series $\sum_n^\infty f_n(x)$ to converge absolutely pointwise on E.

   We remark that, in the case of series convergence, the convergence of the series means the convergence of the sequence of partial sums and the limit (when it exists) is called the sum of the series and written

$$\sum_{n=0}^{\infty} f_n(x) = \lim_{n\to\infty} \sum_{k=0}^{n} f_n(x).$$

Results (Theorems, propositions, lemmas, corollaries):

You should know all of the results we discussed (and especially those we proved) in class. I'll list most of these results, in particular, as they appear in the 2nd edition of Wade's book. Unless otherwise mentioned, you should know the statement of the result precisely and have a really good idea of how they are proved – ideally, you should be able to reproduce the proof.

1. Remark 5.5
2. Remark 5.7
3. Remark 5.8
4. Theorem 5.10
5. Remark 5.14
6. Theorem 5.15
7. Theorem 5.16
8. Theorem 5.18.
9. In line with the above, you should know Darboux's theorem (but you don't need to know how to prove it – we will in MA338):

   Theorem 1. *Let $f : I \to \mathbb{R}$ be bounded on $I = [a, b]$. Then $f \in R(I)$ if and only if, there is a number $\mathcal{I}$ such that, for every $\epsilon > 0$, there is a $\delta > 0$ for which*

$$\left| \mathcal{I} - \sum_{k=1}^{n} f(t_k)\Delta x_k \right| < \epsilon$$

   whenever P is a partition of I with $\|P\| = \max_{k=1,\ldots,n} \delta x_k < \delta$ and the t_ks are admissible for P in the sense that $t_k \in [x_{k-1}, x_k]$ for all $k = 1, 2, \ldots, n$. If either condition is satisfied (and hence both are satisfied), then

$$\mathcal{I} = \int_a^b f(x)\,dx.$$

10. Theorem 5.19. Note, in class, I used Darboux's theorem to do this but either proof is fine.
11. Theorem 5.20. You don't need to know how to prove this, but have a good idea about the proof.
12. Theorem 5.21
13. Theorem 5.22. You should know this proof and also the one I did last week when $f \in R(I; \mathbb{C})$.
14. Corollary 5.23. You don't need to know how to prove this.
15. Theorem 5.24. You should know how to prove this and then also deduce the following corollary from it:
16. If f is continuous on $I = [a, b]$, then there is some $x_0 \in I$ for which
$$f(x_0) = \frac{1}{b-a} \int_a^b f(x)\, dx.$$
17. Theorem 5.26
18. Theorem 5.28
19. Theorem 5.31
20. Theorem 5.34 – you don't need to know how to prove this.
21. Theorem 7.9 (uniform convergence preserves continuity). This is the famous $\epsilon/3$ proof.
22. Theorem 7.10. (You can exchange the order of integrals and limits with uniform convergence)
23. Lemma 7.11 (You don't need to know how to prove this)
24. Theorem 7.12 (You don't need to know how to prove this)
25. Theorem 7.14
26. Theorem 7.15 This is called the Weierstrass M test. We will do it on Monday.

0.1 Things you should be able to do:

1. Compute Riemann sums and upper and lower Darboux sums.
2. One way or another, determine if a (sufficiently simple) bounded real-valued function f is Riemann/Darboux integrable or not. In the case that it is, be able to compute $\int_a^b f$ using upper/lower sums and or limits of Riemann sums. You should be able to run some calculations without making use of the FTC.
3. You should be able to use the FTC, both parts to do calculations.
4. You should be able to employ integration by parts and change of variables (u-substitutions) to compute some simple integrals.
5. You should be able to compute some simple improper integrals.
6. You should also be able to compute the integrals of complex-valued functions on an interval I. E.g., use the techniques in Homework 2.
7. Given a sequence of functions, you should know how to determine if the sequence converges pointwise and/or uniformly and determine the limit. E.g., I worked three examples in class which appear in Remarks 7.3-7.6 in the second edition of Wade and you should know these examples backwards and forwards.
8. You should be able to compute certain series of nice functions and use the results about differentiation, integration to do calculations with them. I will do more in class on Monday/Tuesday.

Things not to worry about:

You know about the following things and why they are important (at least for us) but I won't ask you anything detailed about them.

1. Don't worry about the full scope of improper Riemann integration.
2. You don't need to know the proof of Darboux's theorem – which is long. If you do want to see a proof, email me and I'll send you one.
3. You don't need to know anything about convex functions or functions of bounded variation.