This is an itemized list of topics we've covered in Math 160 since the beginning of the semester. This list of topics (and the proportion of time we've spend on them since the beginning of the semester) will align with the problems you will see on Thursday's midterm exam. In studying for the exam, please note that I consider the homework exercises and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam. For more practice problems, please do additional exercises from the textbook.

Taylor Polynomials (Chapters 1.1-2.3 in Gouvêa)

You should be able to:

1. Find the linear and quadratic approximations of a function at a specific point.
2. More generally, find the nth Taylor polynomial $T_n(x)$ of a function at a specific point.

Sequences (Chapter 11.1)

You should be able to:

1. Define and give examples of sequences of numbers. Also, be able to define and cook up several examples (and non-examples[1]) of:
   (a) bounded sequences
   (b) monotone sequences (eventually monotone sequences)
   (c) non-negative sequences (eventually non-negative sequences)
   (d) convergent sequences
   (e) divergent sequences
   (f) sequences that diverge to $\pm\infty$.
2. Find the terms of a sequence defined by a recurrence relation
3. Determine whether a sequence converges or diverges
4. Be able to show that certain unbounded sequences diverge to $\pm\infty$.
5. State and apply the ϵ-N definition of a limit of a sequence
6. Use the algebra of limits to compute limits of sequences.
7. State and apply the monotone bounded convergence theorem
8. State and apply the squeeze theorem

General Series (Chapters 11.2-11.7)

You should be able to:

1. Given a series $\sum a_k$, define the sequence of partial sums $\{S_n\}$. In particular, you should be able to explain the relationship between the sequence of partial sum and the sequence of terms.
2. Know how to compute partial sums and, in particular, be able to compute them exactly in cases of geometric and telescoping series.

[1] I.E., examples that do not satisfy the condition.

3. Be able to define what it means for a series to converge or diverge based on the behavior of its partial sums. In the case of convergence, be able to define what the sum of the series is. In special cases, e.g., for geometric series, be able to compute lots of examples.

4. For series with non-negative terms, know why (and be able explain) the convergence of series boils down to partial sums being bounded.

5. For series with non-negative terms, be able to explain why the comparison test works.

6. State and apply the following convergence tests:

   (a) Divergence test
   (b) Geometric series test
   (c) Integral test
   (d) p-series test
   (e) Comparison test
   (f) Alternating series test
   (g) Ratio test

7. Compute the value of any geometric series

8. For series with non-negative terms (to which the integral test easily applies) be able to estimate the value of partial sums and, in the case of convergence, sums and remainders.

9. For alternating series that converge, be able to estimate the remainders (i.e., how close each partial sum is from the sum of the series).

10. Be able to determine if a series is convergent and, if so, whether this convergence is absolute or conditional.

Power series (Chapters 11.8-11.10)

1. Given a power series $\sum_n c_n(x-a)^n$, be able to

   (a) Determine its radius and interval of convergence (using, in particular, the ratio test).
   (b) If $R > 0$, be able to determine the convergence/divergence of the series at the endpoints $a \pm R$.
   (c) Be able to determine exactly where a series diverges, converges, and converges absolutely.

2. Be able to manipulate series (using algebra, integration, differentiation of known series) to find a power representation of a known function.

3. For such known functions f (from the previous item) and their associated power series $\sum c_n(x-a)^n$, be able to determine the value of $f^{(n)}(a)$ for any n. For example, could you tell me in three or fewer lines what the 100th derivative of $3x^2/(1-x)^3$ is at $a = 0$? How about the 101st derivative?

4. For an infinitely differentiable function f, be able to compute its Taylor series at at point and determine its radius of convergence. Note: You can think of this as an $n \to \infty$ version of the Taylor Polynomials we studied at the very beginning of the semester.