Math 135: Homework 1

Please complete the following exercises below and write up your solutions consistent with the directions in the syllabus[1]. Your solutions are due on Thursday, September 12th at 10:00AM in the appropriate box outside my office door. If you get stuck on any part of the homework, please come and see me. More importantly, have fun!

Exercise 1 (Introductions)**.** With the aim of getting to know all of you, please answer the following questions.

a. What is your preferred name, i.e., what should I call you in class?

b. What is an interesting fact about you?

c. Why are you taking this course?

d. Beyond mathematics, what subject is most fascinating to you?

e. What was the last math class you took? How did you like it?

f. What role do you think honors calculus will play in your life after this class?

g. Is there anything you wish me to know about you as a student of calculus?[2]

h. Do you have any dog[3] allergies (or any fear of dogs)?

Exercise 2. Argue that $\sqrt{3}$ is not rational. Why does your argument fail when trying to repeat it for $\sqrt{4}$?

Before the following exercises, I'll take the opportunity to introduce some basic mathematical concepts. So, you should read the text below before you tackle any subsequent exercises.

Let A and B be sets. A **function f from A to B** is a rule that assigns each element a of A to a single element b of B and we write $f(a) = b$. To indicate that f is a function from A to B we will often write $f : A \to B$. The **range of f** is the set of those elements in B that can be written in the form $f(a)$ for some a in A. In symbols, this is the subset of B of the form

$$R(f) = \{b \in B : f(a) = b \text{ for some } a \in A\}.$$

Note, as I stated in class, the symbol $\in$ means "lives in" or "is a member of".

The following two properties of functions are important:

1. $f : A \to B$ is said to be **onto** or **surjective** if $R(f) = B$. In other words, every element in B is "hit" by f in the sense that, for each $b \in B$, there is some $a \in A$ for which $f(a) = b$.

2. $f : A \to B$ is said to be **injective** or **one-to-one** if, for each $b \in R(f)$, there is only one element of A for which $f(a) = b$. In other words, f is injective if $f(a_1) = f(a_2)$ can only happen if $a_1 = a_2$.

In the case that f is both surjective and injective, we say that f is **bijective** or the f is a **bijection**. If, given sets A and B, there is a bijection f mapping from A to B, we say that **A and B are in one-to-one correspondence.**

Exercise 3. Consider the sets

$$N = \{1, 2, 3, 4, 5\}, \qquad S = \left\{\text{☺}_1, \text{☺}_2, \text{☺}_3, \text{☺}_4\right\}, \quad \text{and} \quad T = \left\{\bowtie, \text{☹}, \text{☼}, \Upsilon\right\}.$$

In the following, I ask you to "cook up" various examples functions. If no function (with the desired properties exists), please explain why it doesn't.

[1] Now is an excellent time, if you haven't already, to read the syllabus carefully.

[2] e.g. "I cannot see orange, so don't use that marker," or "I have a dreadful fear of trigonometric functions," "I am a visual learner" "I am on the (insert sport here) team," etc.

[3] I ask this because sometimes I bring my dog into office hours and, before I bring her in, I want to make sure that everyone is okay with it.

1. Find a function from N to S which is neither one-to-one nor onto.
2. Find a function from S to T which is neighter one-to-one nor onto.
3. Find a function from N to S which is onto. Is your function also one-to-one? If not, can you find a one-to-one function?
4. Find a function from S to N which is onto. Is your function also one-to-one? If not, can you find a one-to-one function?
5. Which of the sets N, S, T are in one-to-one correspondence? Argue that your assertion is true.

In the following exercise, I will refer to the natural numbers, the positive natural numbers and the integers. While not all authors will agree, my conventions for these are

$$\textit{The natural numbers} = \mathbb{N} = \{0, 1, 2, 3, 4, \dots\},$$

$$\textit{The positive natural numbers} = \mathbb{N}_+ = \{1, 2, 3, 4, \dots\},$$

and

$$\textit{The integers} = \mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, 4, -4, \dots\}.$$

Also, given a positive natural number n, the set of integers from 1 to n is denoted by

$$[n] = \{1, 2, \dots, n\}.$$

For example,

$$[4] = \{1, 2, 3, 4\} \qquad \text{and} \qquad [7] = \{1, 2, 3, 4, 5, 6, 7\}.$$

Exercise 4. In this exercise, you will decide if the given sets are in one-to-one correspondence. So, for each item below, I'll give you two sets. Please decide if they are in one-to-one correspondence and explain your answer carefully, either by "cooking up" a bijection or arguing why a bijection cannot exist.

1. Given some positive natural number $n \in \mathbb{N}_+$, the sets

$$[n] \qquad \text{and} \qquad \{-n, -(n-1), \dots, -2, -1\}.$$

2. Given some positive natural number $n \in \mathbb{N}_+$, the sets $[n+1]$ and $[n]$.
3. The natural numbers $\mathbb{N}$ and the positive natural numbers $\mathbb{N}_+$.
4. The natural numbers $\mathbb{N}$ and the even natural numbers

$$2\mathbb{N} = \{0, 2, 4, 6, 8, \dots\}.$$

5. The natural numbers $\mathbb{N}$ and the integers $\mathbb{Z}$.

Exercise 5. Please do Exercise 4.4 from the "Countability" handout by Leo.

Exercise 6. Please do Exercise 4.5 from the "Countability" handout by Leo.