Course Location: MWF 9 – 9:50 AM   Keyes 102
Professor: Scott Taylor
Email: scott.taylor@colby.edu
Webpage: http://www.colby.edu/personal/s/sataylor
Office Hours: Monday: 1 - 1:50
Tuesday: 2 - 2:50 and 3 - 3:50 (dedicated)
Wednesday: 3 - 3:50 (subject to change)
Thursday: 9 - 9:50 (dedicated)
Friday: 1 - 1:50
And by appointment!
Office: Davis 207
Prerequisites: MA 102, 121, 122, or 161.
Published by Addison-Wesley.
Optional Text: How to Think Like a Mathematician by Kevin Houston.
Published by Cambridge UP.

The Course: MA 274 is a writing course. You will learn how to write a mathematical argument in an accessible and understandable way. You will improve in your ability to devise correct arguments. The skills you develop in this course are essential for all future courses in pure mathematics and many courses in applied mathematics. Furthermore, the ability to devise and structure a mathematical argument using precisely defined technical concepts is a skill that is transferable to other domains, such as computer science and law.

The best way to improve your ability to write and create mathematics is to do it. Thus, this course is structured so that you first encounter new concepts on your own, have the opportunity to wrestle with them, and then go to class where misunderstandings can be cleared up and gaps can be filled. This is certainly the best way to learn this material. At the end of this syllabus, you will find suggestions for how to succeed in this course — I encourage you to contemplate them frequently. We are in this together — you are always welcome to stop by my office with questions or comments. My task is to guide you on your mathematical journey, steer you clear from potholes, and help you keep the end in view.

As with any writing, you need to write about something. In this course, you will be learning and writing about the following concepts: axiom systems, set theory, functions, orders, equivalence relations, cardinality, and graph theory. Each of these concepts is connected to some very beautiful mathematical ideas. We’ll have fun exploring these.
Objectives for increasing mathematical maturity:
By the conclusion of the course students will have improved in their ability to:

- Use definitions and axioms to arrive at correct proofs of theorems by careful logical reasoning.
- Write coherent proofs in a conventional mathematical style
- Prove theorems using direct proofs, proofs by contradiction, proofs by induction, and proofs by contraposition
- Be able to generate examples of or counterexamples to mathematical claims
- Give coherent verbal explanations of mathematics
- Evaluate the correctness or incorrectness of a proof
- Engage in significant self-teaching of mathematics
- Be comfortable when encountering new mathematical concepts
- Understand something of the goals, methods, and culture of several different mathematical disciplines

Specific Course Content Objectives:
The students will study in some detail the following mathematical concepts:

- Formal logic
- Naïve set theory
- Relations, including functions, orderings, and equivalence relations
- Cardinalities of sets
- Axiom systems for various branches of mathematics (for example, group theory and point-set topology)

Evaluation:
The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:
20% minimum of Exam 1, Exam 2
25% maximum of Exam 1, Exam 2
25% Final exam
20% Daily/Weekly homework
5% Journal (completeness) and Ultimate Proofs
5% Class participation including “Proof Projections”

Course letter grades will be assigned (subject to above caveat) according to the following scale. Any curve will be determined at the end of the course, according to the discretion of the instructor.

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<thead>
<tr>
<th>Grade</th>
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<tbody>
<tr>
<td>A</td>
<td>93 - 100 %</td>
<td>C</td>
<td>73 - 77 %</td>
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<tr>
<td>A-</td>
<td>90 - 93 %</td>
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<td>B+</td>
<td>87 - 90 %</td>
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<td>B</td>
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<tr>
<td>B-</td>
<td>80 - 83 %</td>
<td>D-</td>
<td>60 - 63 %</td>
</tr>
<tr>
<td>C+</td>
<td>77 - 80 %</td>
<td>F</td>
<td>below 60 %</td>
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Students who demonstrate an exceptional interest and ability in the course may be given an A+.
**Academic Honesty**

Academic honesty means that the work you present is your own, the ideas you communicate actually represent what you think and feel, and that you are upfront about your sources and inspirations. An act of academic dishonesty can either be intentional or unintentional. In either case, there are both informal and formal consequences. Being scrupulously honest is essential for the functioning of the college, for getting the most out of your own education, and for the success of mathematics as a discipline.

In this course, the greatest temptation towards academic dishonesty is presenting someone else’s proof as your own (perhaps with minor modifications). If someone other than the TA or professor gives you the key idea for a proof, you must credit them in your write-up. *This includes any help you receive from online sources.*

**Exams:**

There will be two midterm exams on the evenings of **March 12** and **April 16** from 7 – 9 PM in a location to be announced. In the event that you are unable to attend the scheduled exam, you must let the instructor know in advance.

The final exam is during exam period **15 on Sunday, May 17** from 6 – 9 PM. It is college policy that the final exam cannot be rescheduled for personal convenience, including airline reservations. In the event that you have three or more final exams scheduled in close proximity, it is sometimes possible to reschedule one or more of them. See the registrar’s webpage for details.

All three exams are cumulative, although the final exam is “more cumulative”. All three exams will present you with new mathematics and will expect you to prove theorems. One of the goals of the course is to equip you to succeed in such a situation. As with homework, your exam grade will be based not only on the correctness of your solutions, but also on your ability to clearly and coherently communicate them.

**Quizzes:**

Periodically there will be short unannounced in-class quizzes. Each quiz will present you with one or more technical terms and will require you to provide the correct definition. The course webpage will keep a running list of terms you are required to know.

**Ultimate Proofs:**

At the end of the semester you will need to select, with input from your professor, two proofs which you have written and which you consider to contain both substantial mathematics and beautiful writing. At the end of the semester, you should give them a final rewrite and put them at the front of your journal.
Class Participation:
This course is designed to teach you to write, discuss, and evaluate new mathematics. You cannot learn to do this unless you are actively involved and participating. It is expected that you will be present in class and will make substantial contributions to class discussion. Since everyone is required to participate it is expected that you will allow your classmates the opportunity to do so and that all classroom interactions will be conducted with decorum and respect.

I strongly suggest that you bring your text and your journal to class each day.

Additionally, each student will be asked to provide a proof for class discussion. These will be scheduled at the start of the semester. Two days before your proof is to be shared, you should meet with me. The final draft will be due via email one day before the proof is to be shared. The proof will be projected onto the screen and the class will discuss its attributes, including ways it might be improved. This will be graded based on how well you meet deadlines and the care with which you write the proof, though not its correctness. Students whose proof projections are due on the same day may not work together on those proofs.

Daily/Weekly Homework:
Daily homework will be assigned and will be collected weekly. All homework will eventually be placed in your journal (see below) and it is expected that whatever you turn in will follow the formatting requirements for the journal (see below).

You may work with other people on the homework assignments, but the write-up must be your own. I suggest that you find a study group and meet regularly with them to discuss the homework. Then, go off by yourself and write up your solutions yourself.

Typically, a daily assignment will consist of reading several sections from the text, solving all or most of the exercises and problems and providing proofs for some of the theorems in the chapter. (This style of teaching is called “modified Moore method”.) Each week you will turn in the daily assignments from the previous week. Assignments should be submitted by placing them in the “official homework binder”. Late assignments may be penalized; please tell me if you are having trouble meeting a deadline.

Note: if you wait until just before you are to turn in the daily assignments to attempt them, you will not succeed and you will not be prepared for class.

In each class meeting, I will assume that you have done the previous daily homework and can discuss it intelligently.

In addition to the part of the daily assignment that is turned in weekly, there will be additional assignments that will be included in the journal, but not turned in. The purpose of these assignments is to help you internalize the definitions and concepts we are learning and to help you be sure you understand the reading.

Parts of an assignment marked (G) are to be submitted in the official homework binder and will be graded. After the graded assignment is returned to you, some problems may be marked with (*). The presence (*) means your solution met a certain threshold of acceptability and that you may redo the problem to earn back any missing points. The acceptable threshold will be determined by the professor or grader and will not be
communicated in advance. The threshold will be based on the quality of the writing and the quality of the mathematics – you should take the problem seriously the first time through it. A rewrite of a problem must be turned in with the next assignment.

**Journal**
The journal will consist a 3-ring binders. It will be collected at every exam. If, at the conclusion of the course, you want it returned to you, please include a mailing address as the first page of the journal.

*Journal Format Overview*
- Your first and last names should be written prominently on the front and side of the binder.
- Nothing should be in the pockets of the binder – every paper in the binder should be 3-hole punched and attached to the rings of the binder. If a paper rips in such a way that it tears loose from a ring, you should fix it in such a way that the paper can turn easily.
- At the top of each page in the binder should be written:
  - Your name
  - Assignment Number
  - The due date of the assignment
  - The part of the assignment that the contents of the page fall under. The required parts of each assignment will be clearly indicated when each assignment is given.

For example:

Scott Taylor
Assignment #2
September 10
Definitions from Sections 1.1 – 1.4

or

Scott Taylor
Assignment #2
September 10
Part 1 (4a): Exercise 1.5.2

- No double-sided page should contain content from more than one part of an assignment or from more than one assignment or should contain the proof of more than one theorem. (Closely related results with short proofs may be on the same page.)
- All assignments, parts of assignments, and exercises must be in the correct order. See below for more details.
- No page should contain prominent erasure marks, cross-outs, writing in the margin, etc. All writing must be completely legible. If you need to rewrite an entire page before submitting your work – do so! Particularly for proofs of theorems, the first draft of a proof is not the one to submit.
Journal Organization:
From front to back, your journal should contain the following. Some of these will not be completed until the end of the semester.

1. Ultimate Proofs (at the end of the semester)

2. The following “Mathematical Engagement” exercises (completed by the end of the semester):
   i. Attending two mathematics department colloquia and writing a one page summary and response. If you are unable to attend the colloquia due to schedule conflicts, see me for alternatives. Before putting the colloquia summary and response in your journal, you should turn it in for comments.
   ii. A one page summary and response for each of the articles distributed in class.

3. Definitions

4. Daily assignments in order, preceded by a print-out of the assignment. Journal problems for each assignment should be placed before the graded problems for that assignment and any rewrites should be placed immediately after the original problem. Journal problems should be in the order in which they were assigned, as should graded problems.

Formatting of journal components
The specific parts of each daily homework assignment will change with each assignment. Here are format requirements for frequently reoccurring parts arising in assignments based on the course text.

Definitions
For each (unless told otherwise) boldface word, write the word, underlined, on its own line followed by its technical definition. You may rephrase the definition from the one given in the text, but what you write must be logically equivalent to the definition given in the text. Between different definitions you should leave one or two blank lines.

For example (from page 69):

Partial Ordering A reflexive, antisymmetric, and transitive relation on a set.

Exercises, Problems, Examples
For each assigned exercise or problem, write the exercise or problem number and the statement of the problem if it is brief. Leave a blank line or two and write your solution. If a problem has multiple parts and the solution to one or more parts takes up a significant amount of space, you may write the statement of each part above the solution for that part, rather than grouping all the statements together. You have permission to be somewhat flexible in the precise formatting of your writing. You should, however, always aim for your work to be so readable that the
grader can read and understand what you have written without referring to the text.

See the guidelines elsewhere in this syllabus for writing proofs. You must write in complete sentences. You may find it helpful to pretend that you are writing a solutions manual for the textbook that will be read by students who are not as capable as yourself.

For example:

1.7.4 Example
Construct converses of the following statements:

1. If Elsie is a cow, then Elsie is a mammal.
2. If \( x = 0 \), then \( x^2 = 0 \).

Solution:

1. If Elsie is a mammal, then Elsie is a cow.
2. If \( x^2 = 0 \), then \( x = 0 \).

Theorems

For assigned theorems write the statement and number of the theorem, leave one or two blank lines, and then provide a complete proof of the theorem. Please see the guidelines on writing proofs for details on my expectations. I will, however, remind you here that all statements (especially mathematical ones!) should be complete sentences. You are certainly welcome to preface your proof with an outline or “sketch” to aid the reader. Again: see the guidelines on writing proofs.

For example:
8.2.2 Theorem (Uniqueness of Inverses)
Let \( x \) be a real number. There exists a unique real number \( y \) such that \( x + y = 0 \). Similarly, if \( x \) is not zero, there exists a unique \( y \) such that \( x \cdot y = 1 \).

Proof:

Claim 1: If \( x \) is a real number then there exists a unique real number \( y \) such that \( x + y = 0 \).

Proof of Claim 1: We must prove both that \( y \) exists and that it is unique. By Axiom 1 (pg 180), there exists an additive inverse \( y \) for \( x \). That is, there exists \( y \) so that \( x + y = 0 \).

It remains to show that \( y \) is unique. To that end, suppose that \( y \) and \( z \) are both inverses for \( x \). By the definition of additive inverse:
\[
\begin{align*}
\text{(a.) } & \quad x + y = 0, \\
\text{(a.) } & \quad x + z = 0,
\end{align*}
\]

Since each thing is equal to itself:
\[
y = y.
\]
Since 0 is the additive identity (Axiom 1), \( 0 + y = y \).

Consequently,
\[
0 + y = 0 + y.
\]
Thus, by equations (a.) we have:
\[
(y + x) + y = (z + x) + y.
\]
Since addition is associative (Axiom 1),
\[
y + (x + y) = z + (x + y).
\]
Consequently, by equation (a.),
\[
y + 0 = z + 0.
\]
Since 0 is the additive identity (Axiom 1),
\[
y = z.
\]

Hence, additive inverses are unique as desired. Q.E.D (Claim 1)

Claim 2: If \( x \) is a non-zero real number, there exists a unique real number \( y \) such that \( x \cdot y = 1 \).

Proof of Claim 2: We must prove both that \( y \) exists and that it is unique. By Axiom 1 (pg 180), there exists a multiplicative inverse \( y \) for \( x \). That is, there exists \( y \) so that \( x \cdot y = 1 \).

It remains to show that \( y \) is unique. To that end, suppose that \( y \) and \( z \) are both inverses for \( x \). By the definition of multiplicative inverse:
\[
\begin{align*}
\text{(a.) } & \quad x \cdot y = 1, \\
\text{(a.) } & \quad x \cdot z = 1.
\end{align*}
\]
Since each thing is equal to itself:
\[
y = y.
\]
Since 1 is the multiplicative identity (Axiom 1), \( 1 \cdot y = y \).

Consequently,
\[
1 \cdot y = 1 \cdot y.
\]
Thus, by equations (a.) we have:
\[
(y \cdot x) \cdot y = (z \cdot x) \cdot y.
\]
Since multiplication is associative (Axiom 1),
\[
y \cdot (x \cdot y) = z \cdot (x \cdot y).
\]
Consequently, by equation (a.),
\[
y \cdot 0 = z \cdot 0.
\]
Since 0 is the multiplicative identity (Axiom 1):
\[
y = z.
\]
Hence, multiplicative inverses are unique as desired. Q.E.D (Claim 2.)

Since we have proven both Claims 1 and 2, QED.
Writing Proofs

The strong proof will exhibit careful logical reasoning combined with succinct expression written with the inexorable drive of compelling mathematical ideas. A proof exhibits “careful logical reasoning” when each statement follows directly and immediately from a definition, axiom, or previously proved result, or a small number of these combined with one or more rules of logic. A proof exhibits “succinct expression” when illustrative examples are not used in place of careful logical reasoning, logical steps considered ‘obvious’ by the reader and which are not necessary, are omitted, and connecting words and transitions between sections while present are as brief and to the point as possible.

Guidelines for a successful assignment write-up¹:

1. Your target audience is your classmates, NOT me or the grader! Your classmates should be able to read and understand your solution without having to ask you any questions about it.

2. You should use only full sentences, mathematical or otherwise. When using mathematical symbols convert them to words in your head to make sure that you are not missing appropriate verbs, articles, etc. Remember that mathematical language has a more rigid format (not unlike a programming language) than written English.

3. Writing many vague or repetitive sentences cannot be a substitute for saying the right thing once. Think your ideas all the way through before you shape them into steps of an argument. Preliminary drafts are strongly recommended!

4. I suggest placing consequent “steps” of an argument on separate lines, similar to the way this is done in instruction manuals, or in presentations of computer algorithms. At the very least, make good use of paragraph breaks.

5. Display longer formulas and diagrams on separate lines the way this is done in textbooks.

6. Skipping steps of an argument or asserting that something is “obvious” (henceforth called a “magic leap”) can be considered a significant or even fatal error. I and the grader are the sole judges of what constitutes a magic leap or an insignificant error, so stay on the safe side and remember part 1!.

7. Do not put the proofs of two different theorems on the same piece of double-sided paper, unless both proofs are short and you are certain they will not need to be rewritten.

8. Your write-up should be laid out carefully and presented in a neat manner. Do a rough draft first, since a sloppy presentation will be penalized. The harder and less pleasant it is for us to follow your argument, the higher are the chances of you receiving a lower grade for the write-up.

¹ Adapted from guidelines used by Leo Livshits.
Other Sources:

- *How to Think Like a Mathematician* by Kevin Houston.
- Pages 1 – 10 of these notes from a class by Donald Knuth: [http://tex.loria.fr/typographie/mathwriting.pdf](http://tex.loria.fr/typographie/mathwriting.pdf)
- Sections 13-16 of this classic article by Paul Halmos: [http://golem.ph.utexas.edu/category/2009/10/halmos_on_writing_mathematics.html](http://golem.ph.utexas.edu/category/2009/10/halmos_on_writing_mathematics.html) (Follow the obvious link to the article.)
- Chapter 0 of *Mathematical Proofs* by Chartrand, Polimeni, and Zhang.

**How to succeed in this course**

MA 274 is an intense course and likely differs considerably from other math classes that you have taken. I recommend the following to help you do your very best in the course and to get the most out of it.

1. **Do some math everyday. Really.**
   Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The homework is intended to help you in your daily practice. But you should spend additional time each day reading the text and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that’s at least 6 – 9 hours per week of homework and studying.

2. **Participate in class. Yes, you.**
   Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what’s going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a question is a great way to learn the right answer. Make an effort to connect each class’s activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can’t do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch. And, finally, volunteer to present in class. This is a great way to get feedback on your writing and proving.
3. **Read the textbook.**
The lectures and the textbook will often present slightly different views on the same material. You will usually read the textbook before discussing the material in class. Think about the difference between the approaches and how they inform each other. After class, reread the section and see how your understanding has changed.

4. **Form a study group.**
Introduce yourself to other people in the class and meet up outside of class to study and work on homework. The fourth floor of Mudd is a great place to meet in the evenings. Be sure that you don’t copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other.

5. **Write your own problems.**
As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems – what makes your problems easier or more difficult than the ones you’ve seen before? Feel free to show me what you’ve done.

6. **Be curious.**
Ask lots of questions. Ask questions to connect our course with material from other courses – especially writing, philosophy, and mathematics courses. Try to predict where the course is going. Revisit material from previous math classes and figure out how it could have been presented if you had taken MA 274 first. Try to create and prove your own theorems. Be curious – this is one of the privileges of being in college.

7. **Visit me in my office.**
I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it’s not my office hours. If I can’t chat, I’ll let you know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event.

8. **Spread the studying out over the semester.**
If you do math everyday, as suggested above, you won’t have to cram for exams. You’ll be able to sleep and, consequently, to think. You’ll be happier and more relaxed. You’ll have time to write papers for your other classes. You’ll have time to appreciate the New England autumn. You don’t need to pull all-nighters.
9. **Have an exam strategy.**

For the in-class exams, you will have only 2 hours in which you have to do some significant mathematics. Be prepared to do some problems very rapidly and be prepared to think about others. If an example or proof was done in class or on homework, you should be able to repeat it very quickly on the exam. Know what you find difficult and what you find easy. Do the easy things first and then the difficult things. Write something for every problem. If you get stuck, tell me how you’d solve it if you could get unstuck. Figure out what the problem is testing and tell me what you know about that area. If the problem is too hard, rewrite it to make it easier. I love to give partial credit. Give me a reason to give you some. If you find yourself getting nervous: breathe deeply, remind yourself you’ve studied thoroughly, then figure out how to do the problem. Keep an eye on the time and don’t spend too long on any one problem.