MA 274: HW #5

Before beginning this homework assignment, please review the guidelines for submitting homework. Remember to write down the total amount of time spent working on the assignment at the top of what you turn in.

1. **SUMMARY OF LOGIC AND PROOF TECHNIQUES**

Read chapter I.2 from *The Princeton Companion to Mathematics* (available through the library catalogue as an ebook or on reserve in the Olin Science Library). Answer the questions distributed in class and turn it in on Wednesday (not Friday!).

2. **A PROOF THAT \( \sqrt{3} \) IS IRRATIONAL**

(1) (G) In this problem you will prove that \( \sqrt{3} \) is an irrational number. You will need the following definition:

**Definition 2.1.** A number \( x \) is **rational** if there exist integers \( a \) and \( b \neq 0 \) such that \( x = \frac{a}{b} \). A number is **irrational** if it is not rational.

Prove that \( \sqrt{3} \) is irrational. When you write up your solution, you should list all elementary number facts that you need (some of which you may not be able to prove.) Any portion of the proof that can be separated out as a separate lemma should be.

(This is your chance to adapt the examples from class to prove something rather sophisticated. You should focus on writing something that is coherently organized and logically correct. At this point you may have gaps in your argument or statements you don’t know how to justify. These should be clearly marked as such. You should base your work on the proof done in class that \( \sqrt{2} \) is irrational. Rather than discussing even numbers, you will need to discuss numbers that are multiples of 3. Also, this is certainly a proof that you can find online – but you shouldn’t do that! Instead work hard to construct it for yourself.)

3. **EVERYTHING YOU WANTED TO KNOW ABOUT SETS, BUT WERE AFRAID TO ASK.**

(1) Read Sections 2.1, 2.2, and 2.3 through page 44.
(2) (J) Write and memorize the definitions of empty set, subset, proper subset, element argument, equality of two sets (Definition 2.2.7).

(3) (G) Prove Theorem 2.2.2 and do Exercise 2.2.4.

(These proofs should be pretty easy. They are assigned to remind you that theorems need proofs and to introduce element arguments.)

(4) (J) Do exercise 2.2.6.

(Actually doing this exercise reinforces the definition of “proper subset”.)

(5) (G) Do exercise 2.3.8