Before beginning this homework assignment, please review the guidelines for submitting homework. Remember to write down the total amount of time spent working on the assignment at the top of what you turn in. These problems give you some exposure to the proper organization and structure of a well-written proof.

(1) (G) Prove that there exists a function \( f : [0, 1] \rightarrow \mathbb{R} \) that is differentiable on \((0, 1)\) such that there does not exist \( x \in (0, 1) \) with
\[
 f'(x) = f(1) - f(0)
\]
(The notation \( f : [0, 1] \rightarrow \mathbb{R} \) means that any number from the interval \([0, 1]\) can be put into \( f \) and then \( f \) will spit out a real number – so this is the kind of function that you’ve often encountered in high school algebra or calculus.

Hint: the function cannot be continuous at \( t = 0 \) or \( t = 1 \) since otherwise the mean value theorem would imply that such an \( x \) does exist.)

(2) (G) Suppose that \( f : [0, 1] \rightarrow [0, 1] \) is a function such that for all \( t \in [0, 1] \)
\[
 \frac{1}{4} t \leq f(t) \leq \frac{1}{2} t.
\]
Prove that there exists a unique \( x \in [0, 1] \) such that \( f(x) = x \). (Recall that you must show that such an \( x \) exists and that it is unique.)

(3) (G) Prove (using a proof by contradiction and some algebra) that the curves in \( \mathbb{R}^2 \) defined by the equations
\[
 x^2 + 3xy + y^2 = 1
\]
and
\[
 y = -2x
\]
do not intersect.