Before beginning this homework assignment, please review the guidelines for submitting homework. Remember to write down the total amount of time spent working on the assignment at the top of what you turn in.

1. **YOU KNOW MY METHODS, WATSON! USE THEM!**

   (1) (J) Find the flaw in the following “proof” that for every non-negative integer $n$ and for every non-zero real number $a$, $a^n = 1$.

   "proof". Let $a$ be a non-negative real number. We prove that $a^n = 1$, for $n \in \mathbb{N} \cup \{0\}$ by complete induction on $n$.

   **Base Case**: $a^0 = 1$

   This is true by the definition of $a^0$.

   **Inductive Step** Assume that $a^j = 1$ for all non-negative integers $j$ with $j \leq n$. We prove that $a^{n+1} = 1$.

   Notice that
   
   $$a^{n+1} = \frac{a^n \cdot a^n}{a^{n-1}}.$$  

   By the inductive hypothesis, $a^n = 1$ and $a^{n-1} = 1$. Thus,

   $$a^{n+1} = \frac{1 \cdot 1}{1} = 1,$$

   as desired.

   □

   (2) (G) Consider a game in which two players (let’s call them Aleph and Bet) take turns removing any number of matches they want from one of two piles of matches. (On their turn, the player can remove matches from whichever pile they want.) Aleph goes first. The player who removes the last match wins the game. Prove that if the two piles contain the same number of matches initially, the second player (Bet) can always guarantee a win.

   (Hint: Prove this using complete induction on $n$, the number of matches in each pile. For the inductive step, when their are $n+1$ matches in each pile, remember that Aleph goes first removing some number (say $k$) matches from one of the piles. Figure out which pile Bet should take matches from and how many matches she should take so that you can apply the inductive hypothesis.)