Before beginning this homework assignment, please review the guidelines for submitting homework. Please write down the total amount of time spent working on the assignment at the top of what you turn in. Remember that at least one problem from each assignment must be written in \LaTeX.  

Remember that if you make use of any sources other than the text or class notes you need to cite it.

You should also start thinking about your final paper. The remaining homework assignments in the course are intentionally smaller in order to give you time to work on it.

1. **Structures on Functions**

In this homework assignment, you’ll explore a little bit concerning how when we have sets with a particular structure we are most interested in functions that preserve that structure.

1. Suppose that $G = (V,E)$ and $G' = (V',E')$ are graphs. A function $f: V \to V'$ is said to be **graphical** if it satisfies the following condition: For every edge $\{v,w\} \in E$, either $f(v) = f(w)$ or $\{f(v), f(w)\} \in E'$. That is, $f$ takes each edge either to a vertex or to another edge. We will often write $f: G \to G'$ if $f$ is a graphical function between their vertex sets.

Recall that $G = (V,E)$ is connected if for every pair $v,w \in V$ there is a path $v_0, v_1, \ldots, v_n$ in $G$ such that $v_0 = v$ and $v_n = w$.

Let $G = (V,E)$ and $G' = (V',E')$ be graphs and suppose that $f: G \to G'$ is graphical. Suppose that $f$ is surjective and that $G$ is connected. **Prove** that $G'$ is connected.

(You might want to start by proving a lemma which says that if $v_0, v_1, \ldots, v_n$ is a path in a graph and if $f$ is a graphical function, then $f(v_0), f(v_1), \ldots, f(v_n)$ has a subsequence with the same starting and endpoints that is a path.)

2. Suppose that $(X,d_X)$ and $(Y,d_Y)$ are metric spaces. Suppose that $f: X \to Y$ is a function with the property: For every $a \in X$, and for every $\varepsilon > 0$, there exists $\delta > 0$ such that if $d_X(a,x) < \delta$ then $d_Y(f(a), f(x)) < \varepsilon$.

Suppose that $(x_n)$ is a sequence in $X$ converging to $a \in X$. Prove that the sequence $(f(x_n))$ in $Y$ converges to $f(a)$. 

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