Before beginning this homework assignment, please review the guidelines for submitting homework. Remember to write down the total amount of time spent working on the assignment at the top of what you turn in.

1. **Reading**

Reread Sections 1, 3, and 4 of [the handout on Equivalence Relations](#) and, additionally, Section 2. This time around pay more attention to the proofs in Section 4.

2. **If it’s all the same to you, we’ll call it an equivalence relation**

   (1) (J) Write and memorize the definition of “equivalence relation”.

   (2) (J) From example 1.2 (in section 1 of the handout), pick 5 of the examples and give a careful proof that they are equivalence relations.

   (3) (J) Do exercise 1.3

   (4) (G) Prove Theorems 1.7 and 1.9. In your proofs you should be sure to specify where you use the various properties of an equivalence relation.

   (5) (G) Consider the equivalence relation $\sim$ on $\mathbb{Z}$ defined by $x \sim y$ if and only if there is an $k \in \mathbb{Z}$ such that $y = x + 7k$.

      (a) List all the elements of $\mathbb{Z}/\sim$. (Hint: you cannot, however, list all the elements of the elements of $\mathbb{Z}/\sim$.

      (b) Prove that if $x \sim y$ then $3x \sim 3y$.

      (c) Does it matter what number is used in place of 3 in the previous part?

      (d) Suppose that $a, b \in \mathbb{Z}$. Define a function $f : [a] \to [b]$ by $f(a + 7k) = (b + 7k)$. Explain why each element of $[a]$ is an acceptable input to $f$. Also show that if $x, y \in [a]$ then $f(x) = f(y)$ implies $x = y$. 