1. Prove that for any odd prime, \( p \),
\[
\frac{p - 1}{2} - \left[ \frac{p}{4} \right] \equiv \begin{cases} 
0 \pmod{2} & \text{if } p \equiv \pm 1 \pmod{8} \\
1 \pmod{2} & \text{if } p \equiv \pm 3 \pmod{8}.
\end{cases}
\]

2. Let \( n = 2^k m \) where \( k, m \in \mathbb{N} \) and \( m \) is odd. Show that \( a \) is a quadratic residue (mod \( n \)) iff \( a \) is a quadratic residue (mod \( 2^k \)) and \( a \) is a quadratic residue (mod \( m \)).

3. Prove that
\[
f(x, y) = x^2 + y^2, \quad g(x, y) = x^2 - y^2, \quad h(x, y) = 2xy
\]
are all inequivalent quadratic forms.

4. Prove that for
\[
f(x, y) = x^2 + xy + 5y^2 \quad \text{and} \quad g(x, y) = 7x^2 + 17xy + 11y^2,
\]
that \( f \sim g \) and find \( U \in SL_2(\mathbb{Z}) \) such that \( f \circ U = g \).

5. a) Compute \( h(-19) \).

b) Show that for
\[
f(x, y) = 5x^2+21xy+23y^2, \quad g(x, y) = 209x^2+247xy+73y^2, \quad h(x, y) = 17x^2+61xy+55y^2,
\]
that \( f \sim g \sim h \).

6. Prove that \(-8 \) is a quadratic residue (mod \( p \)) for a prime \( p \) if and only if \( p \equiv 1, 2, \) or \( 3 \pmod{8} \).

7. Suppose \( n = x^2 + 2y^2 \) for \( x, y \in \mathbb{Z} \) such that \( p \mid n \) with \( p \) a prime where \( p \equiv 5 \) or \( 7 \pmod{8} \). Prove that \( p^2 \mid n \) and that \( n/p^2 = (x')^2 + 2(y')^2 \) for some \( x', y' \in \mathbb{Z} \).

8. a) Compute \( h(-8) \).

b) Use part (a) and Problems 6 and 7 to show that for \( n \in \mathbb{N}, n = x^2+2y^2 \) for \( x, y \in \mathbb{Z} \) iff all primes \( p \mid n \) of the form \( p \equiv 5 \) or \( 7 \pmod{8} \) have even powers in the prime factorization of \( n \).
9. a) Compute $h(-7)$.

b) Show that any prime $p = x^2 + xy + 2y^2$ for some $x, y \in \mathbb{Z}$ iff $p \not\equiv 3, 5, 6 \pmod{7}$.

10. Let the prime $p$ be such that $p \equiv 7 \pmod{8}$. Show that

$$p = x^2 + y^2 + z^2$$

has no solutions for $x, y, z \in \mathbb{Z}$. 