Examples:  a) \((3+2i)+(6-3i) = (3+6) + i(2-3) = 9-i\)

\[ b) \ (4+i\pi)(\sqrt{2}+3i) = (4\sqrt{2}-3\pi) + i(12+7i) \]

\[ c) \ \frac{6+2i}{5-i\sqrt{3}} = \frac{(6+2i)(5+i\sqrt{3})}{(5-i\sqrt{3})(5+i\sqrt{3})} = \frac{(30-2i\sqrt{3})}{28} + i\left(\frac{10+6i\sqrt{3}}{28}\right) \]

§1.2 Point representation of complex numbers

Recall the Cartesian coordinate system:

If I have the point \((2,3)\) I plot it as such

\[-2+3i = (2,3)\]

<table>
<thead>
<tr>
<th>-2</th>
<th>-1</th>
<th>-1</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>3</td>
</tr>
</tbody>
</table>

We can plot complex numbers similarly, like \(3-3i\)

Where we plot the real part along the \(x\)-axis (Real axis)

The imaginary part along the \(y\)-axis (Imaginary axis)

Definition 3: The absolute value or modulus of the number \(z = a + bi\) is denoted by \(|z|\) and is given by \(\sqrt{a^2+b^2}\)

So \(|z|\) is the distance of the point \(z\) from the origin (Zero) on the complex plane
Furthermore, if $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$, then
\[ |z_1 - z_2| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} \]
which is the distance between $z_1$ and $z_2$.

**Examples**

a) Illustrate the set of points $z \in \mathbb{C}$ that satisfy
\[ \text{Im} \ |z - (2 - i)| = 2 \]

A circle of radius 2 centered at $2 - i$.

In general, $|z - z_0| = \Gamma$ describes a circle of radius $\Gamma$ centered at $z_0$.

b) Illustrate the set of points $z \in \mathbb{C}$ that satisfy
\[ |z - 1| = |z - i| \]
6. Two ways of thinking about this:

**Geometrically:** the set of points \( z \) that are equidistant from 1 and \( i \):

\[ \text{Im} \]

\[ \text{Re} \]

This line, \( y = x \)

**Algebraically**

\[ |x + iy - 1| = |x + iy - i| \]
\[ \Rightarrow \sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y-1)^2} \]
\[ \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2 \]
\[ \Rightarrow 2x - 2y = x^2 + y^2 - 2y + 1 \]
\[ \Rightarrow -2x = -2y \]
\[ \Rightarrow x = y. \]

C) \( |z + i| = \text{Re } z + 2 \)

Geometry is less obvious here (unless you’ve reviewed conics) so we do the algebraic interpretation.

\[ \sqrt{x^2 + (y+1)^2} = x + 2 \]
\[ \Rightarrow x^2 + (y+1)^2 = (x+2)^2 \text{ and } x = -2 \]
\[ \Rightarrow x^2 + (y+1)^2 = x^2 + y^2 + 4 \]
\[ \Rightarrow y = \frac{(y+1)^2 - 1}{x+2} \]

Parabola

(SET OF POINTS EQUIDISTANT FROM \(-i\) AND THE LINE \( x = -2 \))
\textbf{Definition 4}: A complex conjugate of \( z = a + ib \) is denoted \( \overline{z} \) ("zed bar") and is given by \( z = a - ib \).

Geometrically this is given by reflection across real axis:

\[ \overline{z} = \overline{a + ib} = a - ib \]

It is easy to check that
\[ (z \pm w) = \overline{z} \pm \overline{w}, \quad (zw) = \overline{z} \overline{w} \quad \text{and} \quad \left( \frac{z}{w} \right) = \frac{\overline{z}}{\overline{w}} \]

Indeed, let \( z = a + ib, \quad w = x + iy \) then

\[ \overline{zw} = \overline{(a + ib)(x + iy)} = (ax - by) + i(by + ax) = (ax - by) - i(by + ax) = (ax - by) + i(-b)x + a(-iy) = (\overline{a - ib})(\overline{x - iy}) = \overline{z} \overline{w}. \]

Furthermore \( \overline{\overline{z}} = z \), also:
\[ \frac{z + \overline{z}}{2} = \frac{a + ib + a - ib}{2} = a = \text{Re}(z) \]
\[ \frac{z - \overline{z}}{2i} = \frac{a + ib - a - ib}{2i} = b = \text{Im}(z) \]

Also \( |z| = |a + ib| = \sqrt{a^2 + b^2} = \sqrt{a^2 + (-b)^2} = |a - ib| = |\overline{z}| \)

And \( z\overline{z} = (a + ib)(a - ib) = a^2 + b^2 = |z|^2 \in \mathbb{R} \)

Not surprising since this is how we "rationalize denominators."
\[ \frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{|z_2|^2} \text{ is real denominator.} \]

\section*{1.3 Vectors And Polar Forms}

We can think of complex numbers as points on the plane, and so we can also think of them as vectors.

And since we already saw that that addition is coordinate-wise, we have that the addition of complex numbers is the addition of vectors. Also the length of the vector is just the magnitude $|z|$.

These observations give us the **triangle inequality**:

\[ |z_1 + z_2| \leq |z_1| + |z_2| \]

Because the length of a side of a triangle is no greater than the sum of the lengths of the remaining side.

Can also see:

\[ |z_1| + |z_2| - |z_1| \geq |z_1 + (z_2 - z_1)| = |z_2| \]

Or

\[ |z_2 - z_1| \geq |z_2| - |z_1|. \]