When necessary, write in symbolic expansion. That is, we would say \((3 + \pi)\) not \(6.14\ldots\) 
Please simplify as much as you can.

For many of these, Wolfram Alpha is a good resource for checking your answers, but it doesn’t explain your steps. If you want credit you need to show your reasoning.

1. (Section 3.1) Give the partial fractional expansions of the functions below.
   
   a) \(f(z) = \frac{2 + i}{(z^2 + 4)}\)
   
   b) \(g(z) = \frac{z - 3}{(z - 1)^2}\)
   
   c) \(h(z) = \frac{4z^3}{z^4 - 1}\).

2. (Section 3.1) If \(z_0\) is a pole of the rational function \(R(z)\) then we call the coefficient of \(1/(z - z_0)\) in the partial fraction expansion the residue of \(R(z)\) at \(z_0\). For example if
   
   \[
   R(z) = \frac{2z^2 - 4z + 1}{(z - 2)^2(z - 1)} = \frac{-1}{z - 1} + \frac{1}{(z - 2)^2} + \frac{3}{z - 2}
   \]
   
   then the residue of \(R(z)\) at 1 is \(-1\) and the residue of \(R(z)\) at 2 is 3.

Find the residue at \(2i\) of

\[
S(z) = \frac{1}{(z - 2i)^3(z + 1)}.
\]
3. (Section 3.2) Let $S$ be the region of $\mathbb{C}$ bounded above by the line $y = x + 2\pi$ and below by the line $y = x$ with the lower boundary included and the upper boundary excluded as in the picture below. Describe the range of $S$ under $f(z) = e^z$ (a picture may be helpful). Is $f$ one-to-one on this set?

![Diagram of region S]

4. (Section 3.2) Show that for $z_1, z_2 \in \mathbb{C}$ we have

a) $\sin(z_1 + z_2) = \sin(z_1) \cos(z_2) + \sin(z_2) \cos(z_1)$  
   b) $e^{z_1 + iz_2} = e^{z_1}(\cos(z_2) + i \sin(z_2))$

[Note: We know these identities if $z_1, z_2$ are real. I want you to prove them for complex numbers.]

5. Compute the following:
   
   a) $\log(e + ie)$  
   b) $\Log(-1 - i\sqrt{3})$  
   c) $\mathcal{L}_{-\frac{\pi}{4}}(-4i)$

6. (Section 3.3) Find the region, $R$, where $f(z) = \mathcal{L}_0(2z^2 + 1)$ is analytic, describe it and sketch a picture. Compute $f'(z)$ on $D$. Is $R$ a domain?

7. (Section 3.3) (Section 3.3) Find a branch of log, $\mathcal{L}_\tau$, such that $f(z) = \mathcal{L}_\tau(z^2 + 1)$ is analytic at $z = 2i$ and $f(2i) = \Log(3) + 3\pi i$. 

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