When necessary, write in symbolic not decimal expansion. That is, we would say \((3 + \pi)\) not \(6.14\ldots\)

*Please simplify as much as you can. Show your work.*

1. (Section 2.3) a) Write

\[
f(x + iy) = (x^2 - y^2 - x) + i(2xy + y)
\]

in terms of \(z\) and \(\bar{z}\). Simplify as much as possible, the answer shouldn't look too complicated.

b) Use part (a) [that is, do not use the Cauchy-Riemann Equations] to show that \(f(z)\) is nowhere differentiable.

2. (Section 2.3) Compute the derivatives, \(\frac{d}{dz}\), of the following functions. Show steps.

a) \(z^n\), where \(n\) is a negative integer

b) \(\frac{z(z + 3i)^47}{(z^2 + 2)^{12}}\)

3. (Section 2.4) Find the the set of points \(\mathbb{C}\) where the following functions are complex differentiable. Are they analytic anywhere?

a) \(f(x + iy) = (1 + i)(x^2y^2)\)

b) \(f(x + iy) = x^3 + i3y\)

4. (Section 2.4) Show that if \(f(z)\) is purely imaginary and analytic on the domain (open, connected set) \(D\) then \(f(z)\) is constant.

5. (Section 2.4) Take it as known that the function

\[
f(z) = e^{-2xy}[\cos(x^2 - y^2) + i\sin(x^2 - y^2)]
\]

is entire and find it's derivative.