Topics in Algebra — Scott Taylor

Syllabus

What and Where

Course: MA 434: Topics in Algebra, Geometric Group Theory

Instructor: Scott Taylor

Email: sataylor@colby.edu – Don't hesitate to email me.

Office: Davis 207.

Office Hours: See personal webpage http://www.colby.edu/ sataylor

Course time: TR 9:30 - 11:15.

Course Location: Davis 217.

Course Texts: *Office Hours with a Geometric Group Theorist* (ed. Clay and Margalit) Princeton UP.
Groups, Graphs, and Trees (Meier) Cambridge UP.
Online versions of these texts are available from the library at the following links. If you are off-campus you may need to use VPN to access.

- Clay/Margalit: https://cbbcat.net/record=b6440070 S19
- Meier: https://cbbcat.net/record=b4825059 S19

Course Materials: Located on the course webpage (`colby.edu/~sataylor`).

Office Hours: I hope you come talk with me frequently about course material! Available times are listed on my webpage. Additonal times are available by appointment.

Academic Integrity

Academic Integrity is essential to the pursuit of truth and to gaining the most out of your studies. It is much more than just the absence of dishonesty. It is giving credit to all whose work you rely on; it is putting forth a good faith effort in the class; it is reaching out for help or encouragement or advice when you need it. It is doing the best you can in the circumstances you find yourself in. I value academic integrity greatly and expect you to. Don't hesitate to start a conversation with me about it. It's a passion of mine. (And, yes, there are consequences for academic dishonesty. It doesn't matter what they are. What matters is if you know the difference between academic honesty and academic dishonesty and choose honesty. If you don't know the difference or if you are unsure – ask! If you need help – ask!) In our course, this plays out in two main ways: If someone (whether another Colby student or someone back home or someone on the internet) helps you out, give them credit. When you rely on other sources, whether books, articles, or websites – give them credit.

About the Course

Geometric Group Theory (GGT) is a relatively new mathematical discipline that blends algebra, geometry, and topology. Unlike in most undergraduate algebra courses, GGT is primarily interested in *countably infinite* groups. Such groups naturally arise as the symmetry groups of infinite spaces. It's a beautiful subject that intertwines algebraic calculation and geometric intuition. Not only is it an exciting new area of research that has attracted many prominent mathematicians and their students, but it also has started to find a variety of applications, including in political science.

This is a 4xx-level capstone course for a major in mathematics or mathematical sciences. It will require substantial effort and involvement on your part, but it will be worth it! In this course, you'll see many themes from your undergraduate education come together and you'll be treated as near equal partners in the mathematical enterprise. Most classes will involve no or minimal lecturing on my part: instead they will be a a blend of in-class projects and student presentations.

Objectives for increasing mathematical maturity:

- Ask questions of varying degrees of sophistication about mathematical concepts, both concrete and abstract.
- Turn informal or imprecise ideas into precise mathematical concepts
- Tackle mathematical problems of unknown difficulty with creativity and courage
- Listen carefully to others discussing mathematics and ask constructive questions
- Give coherent written and verbal explanations of mathematics
- Engage in significant self-teaching of mathematics
- Be comfortable when encountering new mathematics
- Be able to express technical ideas in both formal and informal ways

Other resources

Software: At various times you may like to use LaTeXor SageMath.
Other texts: You may at times find it helpful to consult undergraduate texts in abstract algebra. I'm particularly fond of Michael Artin's *Algebra*, but there are many others.

Course Structure

As this is a 4xx-level seminar, you and your classmates will be responsible for a large percentage of how we use class time. In general, the course will be structured this way:

- Tuesdays: You will have a **very short** assignment due in class. Class time will be spent with in-class projects and me explaining math to you.
- Thursdays: Two students will be in charge of presenting material and two students will be in charge of asking questions.

Assignments, Grading and such

Grading: Grading serves three main purposes – it provides extrinsic motivation for you to do important work, it enables me to give you feedback on your work, and it helps me to assign a rough marker as a measure of my estimate of your mastery of course material. If I have the sense that everyone devoting themselves to the course, is coming to class, is turning work in on time, is prepared for their presentations, is attentive to their classmates' presentations then the grading scheme will be:

- (25%) attendance, weekly assignments and in-class projects, graded for completeness only.
- (70%) presentations and question-asking (see below for how these will be graded).
- (5%) Final reflection paper, in lieu of a final exam/project.

If on the other hand, presentations are sloppy or unprepared, then we may renegotiate the grading scheme, perhaps by implementing weekly quizzes to help everyone stay on track.

However, **the pace of the course is flexible and adaptable.** Depending on background, interest, and intrinsic aptitude different students learn math at different paces. As long as you are committed to the course, the course will adapt to where you are. **Homework:** A short HW assignment will be collected every Tuesday. It will generally consist of a few questions intended to help you review what has recently been covered and will have a practice problem or two.

Presentations and Questions, Big Picture: The presentations and questions will be assigned on a rotating basis. Assuming that we have 5 students in the course (as of time of writing) this means that you will present once every 3 weeks, be in charge of questions once every 3 weeks, and have no special responsibility once every 3 weeks. Two students will present every Thursday. Your presentation should last up to 30 minutes (with no interruptions) and will closely hew to an assigned section of one of the textbooks, but with "added value," often taking the form of additional pictures or examples. Often some minimal coordination between the presenters will be helpful, but for the most part the presentations will be separate from each other. More detail on presentations is below. The students in charge of asking questions will have read the material in advance of the class and will ask a mix of prepared and spontaneous questions. The prepared questions will be turned into me on or before class time. The presenter does not need to be able to provide correct answers to the questions, but they should make a good faith effort to think about the questions and attempt to answer it. After they've done so, other students or I will discuss possible answers. The goal of the questions is to help ensure that everyone in the class is understanding the presentation and to spark mathematical conversation. The person who does not have a special responsibility may also ask questions.

Suggestions for preparing a presentation:

(1) Read the assigned section for both you and your fellow-presenter, skipping technical details. Identify the main point of your section (often a theorem statement or the construction of an example). Identify two or three subsidiary points.

(2) Ask yourself: How is the section related to what came before? How is it related to what came after.

(3) Read it again, this time go for the next level of detail. As you work through the details, have a running example or draw additional pictures to help you understand. Read with a skeptical mind: ask yourself if what the author is saying is actually true or if there is a different approach that might be clearer.

(4) Identify technical details that you don't understand and ask for help.

(5) Identify what technical details you should not present. Be sure, however, that your presentation has real content to it.

(6) If they exist, look at exercises in the book pertaining to your section. Do one or two of them and use them as a source for examples and a way of testing your understanding.

(7) Ask yourself: "what value can I add to what's in the book?" and then figure out how to add it!

(8) Write your presentation.

(9) Practice your presentation, out loud, on a board.

Suggestions for what makes a good presentation: These are suggestions. Depending on what you are presenting, you may intentionally choose to ignore or adapt these.

(1) Start with a motivating question.

(2) State the point of your presentation.

(3) Remind us of important definitions or relevant earlier results.

(4) Give an outline of your talk or the proof you are presenting, focusing on the high-level points.

(5) Introduce a running example or draw pictures to help understand how ideas fit together

(6) Distinguish between medium-level details and very technical details. You may choose to omit some technical details, but be sure to say what you've omitted. Be honest and forthcoming when there's something you are unsure about or don't understand.

(7) Conclude by reiterating the main point.

(8) Draw lots of pictures or give lots of examples.

(9) Write a lot on the board, but don't write everything. If you have complicated pictures, prepare them beforehand and project to the screen.

(10) Organize your board work: use as much of the board as possible, work from left to right, use vertical separating lines and headers for new sections.

(11) When someone asks a question, take a moment to think before responding. If you don't know the answer, feel free to speculate but make it clear that's what you are doing.

How to prepare questions:

(1) Read the assigned sections in advance and identify the main point and two or three subsidiary points for each.

(2) Ask yourself, where is the author being vague or imprecise? Are there any steps that were skipped?

(3) Think in terms of:

- **Clarification Questions.** These are questions arising from confusion or unclarity about what is being said. For example: "Why does the definition of the derivative contain a limit?"

- **Digging Deeper Questions.** These are questions intended to push an underlying mathematical idea to the next level. For example: "What would go wrong if we define the derivative of f at a as $\lim_{x\to a}(x-a)/(f(x)-f(a))$ instead of $\lim_{x\to a}(f(x)-f(a))/(x-a)$? What would it measure?"

(4) Write down one or two clarification questions pertaining to the parts that seem most difficult.

(5) Write down digging deeper questions, perhaps by asking about the possibility of dropping hypotheses or trying to get a stronger result.

(6) As one of the assigned question-askers, you have primarily responsibility for politely interrupting the presenter to ask for more detail or for clarification if either you are not understanding something or if you feel a classmate may not understand something.

Expectations for the "off day": On a Thursday when you are neither presenting nor in charge of asking questions, here is what is expected:

(1) You read the assigned sections, skipping all technical details but making note of a few major points.

(2) You have reviewed what was done in previous classes.

(3) You have made note of what in the course you are understanding well and what you are not. You have sought help from the professor concerning major concepts you are not understanding.

Health: You are expected to follow College health protocols (e.g. masking requirements) at all times. If you are sick, especially with any Covid-like symptons, or if you have a positive Covid test, you may not attend class. If this happens, let me know ASAP before class and I will arrange for you to join via Zoom or to have class recorded. If you are scheduled to present, you may either give a virtual presentation (if you are not very sick) or record a presentation to make up for the missed presentation after-the-fact. If you are scheduled to ask questions and are too sick to join virtually, you should email me your questions in advance of class.

Other useful things:

Religious Holidays Colby College is supportive of the religious practices of its students, faculty, and staff. The College is committed to ensuring that all students are able to observe their religious beliefs without academic penalty.

The College will enable any student to make up any course requirements scheduled during a religious holiday that is observed by that student. Students are expected to inform course instructors within two weeks of the beginning of the term of any religious observance that will conflict with coursework. The faculty member will then work with the student to find a reasonable accommodation that will allow the student to complete the academic work. In addition, no student will be required to participate in college events such as athletic commitments, lectures, or concerts on these holidays.

Personal Note: As a practicing Christian, I attempt to avoid working on Sundays. Emails I receive on Sunday are generally not answered until Monday. Not only is this a personal religious practice, it is an aspect of healthy living. I encourage you to establish, as much as possible, your own day of rest. If your schedule or workload does not allow you to set aside an entire day, I encourage you to be intentional about reserving a contiguous period of several waking hours for media-free rest, self-reflection, service or worship.

Learning Differences: Students with learning differences are encouraged to meet with me to discuss strategies for success. I am committed to helping all students succeed and to making reasonable accommodations for documented learning differences.

Sexual Misconduct/Title IX Statement: Colby College prohibits and will not tolerate sexual misconduct or gender-based discrimination of any kind. Colby is obligated to investigate sexual misconduct (including, but not limited to sexual assault and sexual harassment). If you wish to speak confidentially about an incident of sexual misconduct, please contact Colby Counseling Services (207-859-4490) or the Director of the Gender and Sexual Diversity Program, Emily Schusterbauer (207-859-4093). Students should be aware that faculty members are considered responsible employees; as such, if you disclose an incident of sexual misconduct to a faculty member, they have an obligation to report it to Colby's Title IX Coordinator. Disclosure may include communication in-person, via email/phone/text, or through class assignments. To learn more about sexual misconduct or report an incident, visit http://www.colby.edu/sexualviolence/.