Topics in Algebra

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## HW 3

Read Clay-Margalit Chapter 2. Some of this was already covered by the reading in Meier, so it's not as bad as it seems.
(1) Let $\Gamma$ be the cubical lattice graph. This is the graph in $\mathbb{R}^{3}$ with a vertex at every point $(a, b, c)$ with $a, b, c \in \mathbb{Z}$ and with edges consisting of all the points with at least two coordinates integers. Give three examples of nontrivial groups that act on this graph. Be sure to state not only what the group is, but also how it acts on $\Gamma$. Can you find a nonabelian group that acts on $\Gamma$ ?
(2) Consider Examples 7 and 8 on page 36. Draw some examples of graphs, choose some points on them, and calculate the distance between them.
(3) Do Exercise 20 for the groups $D_{8}$ and $S_{4}$. You can choose the generating sets, as long as you don't choose the generating set to be equal to the whole group.
(4) Consider the examples of group actions by isometries given on page 39. Find two more examples and explain them.
(5) (Bonus, not required) We discussed Theorem 2.3 in class but we define the centroid of a finite set of points in $\mathbb{E}^{n}$ to be the result of adding their coordinates and dividing by the number. The book defines the centroid to be the point that minimizes the total distance to the points in the set. Prove the definitions are equivalent.

