Topics in Algebra — Scott Taylor

HW 1

Read Clay-Margalit Sections 1.1, 1.2, 1.3. Section 1.1 is the most important for now. For the others, just concentrate of identifying what's new and what's review.

(1) On page 6 of the reading, the authors explain that the reflections s and t generate D_5, the set of symmetries of a regular pentagon. For each of the following, explain your answers using a general argument, not by actually doing 20 calculations each. You can certainly use pictures to help explain.

   (a) Find a rotation and reflection of a regular decagon (10-sided polygon) that generate D_{10}.

   (b) Find two reflections of a regular decagon that generate D_{10}.

   (c) Find two reflections of a regular decagon that **do not** generate D_{10}.

(2) In the permutation group S_8, do the following computations. For each, draw a "braid diagram" indicating how the composition works.

   (a) $(135)(345)$

   (b) $(1234)(18)(3452)$

(3) For your Archimedean solid, let G be its orientation-preserving symmetry group. To write up your solutions, you may want to take and edit photos of your object.

   (a) Find the smallest set of generators you can for G. (You do not need to prove your set is the smallest possible.)

   (b) Show that G is non-abelian.

   (c) Use the orbit-stabilizer theorem to compute $|G|$.