MA 262: Review 3 **Name:**_____

This review concerns the mean value theorems for integrals and derivatives. Here are there statements:

Theorem (MVT for Derivatives)**.** Suppose that $f\colon [a,b] \to \mathbb{R}$ is continuous and that it is differentiable on (a,b). Then there exists $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

Theorem (MVT for Integrals)**.** Suppose that $f\colon [a,b] \to \mathbb{R}$ is continuous. Then there exists $c \in [a,b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

(1) Let $f(x) = x^2$ for $x \in [0,2]$. What does the mean value theorem for derivatives say when applied to f?

(2) Let $f(x) = x^2$ for $x \in [0,2]$. What does the mean value for integrals say when applied to f?

(3) What does the mean value theorem for integrals say about the expression:

$$\frac{1}{b-a}\int_a^b g(x)\,dx?$$

(4) What does the mean value theorem for derivatives say about the expression:

$$\frac{g(2,3)-g(2,1)}{3-1}$$

assuming that $g(x,y)$ is C^1 at all $(x,y) \in \mathbb{R}^2$ such that $0 \le x \le 2$ and $1 \le y \le 3$?