MA 262: Review 2 **Name:**_____

This review concerns the notion of "differentiability" and asks you to remember the mean value theorem from Calculus 1. You may wish to review this concept in a Calculus 2 book or on pages 97 - 124 of our text. Answer these questions on a separate sheet of paper.

(1) What are the intuitive and rigorous definitions of limit of a function $f\colon \mathbb{R}^n \to \mathbb{R}^m$ at a point $\mathbf{a} \in \mathbb{R}^n$? (Definitions 2.1 and 2.2 in the text) How do the εs and δs make the intuitive definition precise?

(2) What is the **definition** of differentiability at a point $\mathbf{a} \in \mathbb{R}^2$ for a function $f\colon \mathbb{R}^2 \to \mathbb{R}$?

(3) It is a fact that simply having partial derivatives at a point $\mathbf{a} \in \mathbb{R}^2$ is not enough to guarantee that a functon $f\colon \mathbb{R}^2 \to \mathbb{R}$ is differentiable there. What additional conditions on the partial derivatives suffice? (Hint: See Theorem 3.5 on page 122)

(4) Use the previous problem to explain why $f(x,y) = \cos(x^2 y)$ is differentiable at every point of $\mathbb{R}^2$.

(5) What does the Mean Value Theorem (from Calculus 1) say? Based on the lecture in class (or Section 2.3 in the text), summarize it's relevance for deriving the arc length formula for C^1 curves.