MA 262: Review 1 **Name:**______________________________

This review concerns the notion of linear (or affine) approximation. You may wish to review this concept in a Calculus 1 or 2 book before attempting these problems. Answer these questions on a separate sheet of paper.

(1) Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin x$.

(a) Find the equation of the line tangent to the graph of f at the point $(\pi/4, \sqrt{2}/2)$.

(b) Find a linear (technically "affine") function $L\colon \mathbb{R} \to \mathbb{R}$ such that L is a "good approximation" to f near $x = \pi/4$.

(c) Use your answer to the previous part to find an approximation to $\sin(\pi/5)$, without using a calculator.

(2) Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable at a point $a \in \mathbb{R}$. What is the general equation of the linear (or affine) approximation to f at $x = a$?

(3) Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = ye^x$.

(a) Find the equation of the plane tangent to the graph of f at the point $(1,2)$.

(b) Find a linear (or affine) function $L\colon \mathbb{R}^2 \to \mathbb{R}$ such that L is a "good approximation" to f near $(x,y) = (1,2)$.

(c) Use your answer to the the previous part to find an approximation to $2.3e^{1.0005}$.

(4) What is the general form of a linear approximation to a differentiable function $f\colon \mathbb{R}^2 \to \mathbb{R}$ at a point $(a,b) \in \mathbb{R}^2$?