

MA 262: Potential Final Exam Questions from Projects (Day 1)

(I) Green's identity and vortex atoms

(a) Prove Green's first identity:

$$\iiint_D \nabla f \cdot \nabla g \, dV + \iiint_D f \cdot \nabla^2 g \, dV = \oint_C f \nabla g \cdot d\mathbf{S}$$

from the identity

$$\iiint_D \nabla f \cdot \nabla g \, dV = \iiint_D \left(\frac{\partial f}{\partial x} \cdot \frac{\partial g}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial y} + \frac{\partial f}{\partial z} \cdot \frac{\partial g}{\partial z} \right) dx dy dz$$

(b) If a vector field, let us call it F , located in a region D and bounded by ∂D is "incompressible", use the 3-D divergence theorem (Gauss' theorem) and the Laplacian operator to prove that the flux throughout the region equals 0.

(c) How did Thompson resolve the issue of multiply connected surfaces?

(II) Green's Theorem and improper integrals

(a) Give an example of an improper integral:

(b) What is one use of Green's theorem?

(c) The Green's Theorem method is a modification of what other method of solving improper integrals?

MA 262: Potential Final Exam Questions from Projects (Day 2)

(III) Cohomology

- (a) Why is it helpful to know the dimension of the cohomology group $H^0(U)$ for some region U ?
- (b) Why is it helpful to know the dimension of the cohomology group $H^1(U)$ for some region U ?
- (c) Name one theorem that can be proved using cohomology.

(IV) Lengths of Level Curves

- (a) What is the definition of a level curve of a function $f(x,y)$ at a real number k ?
- (b) Regarding the divergence-derived definition of the length of a level curve for a function f at a particular value: What is the vector field of which we find the divergence?
- (c) In addition to Green's Theorem, what other essential mathematical theorem is utilized in proving the definition of the length of a level curve?

MA 262: Potential Final Exam Questions from Projects (Day 3)

(V) Gravity

- (a) According to the Shell Theorem, what would the gravity be at the center of a shell with constant density and mass M ?
- (b) What law is very useful to use when it comes to proving the Shell Theorem?
- (c) Why is the Gaseous Model useful?