Spring 2019/MA 262 **HW 3: If you get an undeserved speeding ticket, is it a moving frame?**

3.A Find integrals in 1 variable equal to the lengths of the following curves. You do not need to solve the integrals.

(1) $f(t) = (t, t^2, t^3)$ for $-1 \le t \le 1$.
(2) $g(t) = (\sqrt{t}, \cos t)$ for $t \in [2, 7]$.

3.B Let $\mathbf{x}(t) = (t^3, 2t^3 + 2)$ for $1 \le t \le 2$. Reparameterize $\mathbf{x}$ by arc-length.

3.C Let $\mathbf{x}\colon [0, b] \to \mathbb{R}^2$ be a parameterized curve. In class, we learned how to reparameterize $\mathbf{x}$ to become a curve $\mathbf{y}$ so that on the interval $[0, t]$, the curve $\mathbf{y}$ has length t. Let $k > 0$ be constant. Show how to reparameterize $\mathbf{x}$ to a curve $\mathbf{z}$ so that on the interval $[0, t]$ the length of $\mathbf{z}$ is kt.

Hint: Start by reparameterizing $\vec{x}$ to $\mathbf{y}$. How do you need to scale the input to $\mathbf{y}$ so that instead of "distance equalling time", "distance equals k times time"?

3.D Let G be the graph of a function $y = f(x)$ in the plane (with f differentiable). A circle of radius ρ is on top of and tangent to G and rolling along it (from left to right) at a rate of 1 revolution per second. At time 0, the circle is tangent to $(0, f(0)$. Let P be a point on the circle. (Which point you choose is up to you.) The purpose of this problem is to find a parameterization $\mathbf{p}(t)$ for the path taken by P.

You will not be able to find a "closed form" solution – your answer will likely have inverse functions and possibly integrals in it (at least implicitly). But, however you write your answer the reader should, in principle, be able to trace everything back to the function f. The problem is essentially asking for a recipe for how to figure out a parameterization of $\mathbf{p}(t)$. You don't need to actually follow the recipe, just give me enough details so that if I knew what f was, I could follow your recipe. Capice?

(1) Find a formula for the position $\mathbf{p}(t)$ of P in tangent space coordinates based at the center $\mathbf{c}(t)$ of the circle.

(2) After t seconds, how far along G has the circle rolled?

(3) Find a parameterization $\mathbf{x}$ of G so that at time t, the circle is tangent to $\mathbf{x}(t)$. (Hint: Use problem C above)

(4) Write $\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$. Find a vector perpendicular to $\mathbf{x}'(t)$ that (when based at $\mathbf{x}(t)$) points above G (rather than below). (Your answer will depend on x and y.)

(5) Find a formula for the center $\mathbf{c}(t)$

(6) Find coordinates for $\mathbf{p}(t)$ in the usual coordinate system. (Your answer will likely have an inverse function in the expression – you won't be able to get it in closed form.)

3.E Use Mathematica to plot an interesting looking vector field. Plot it so that all the arrows are the same length. You may make up your own equations. The only rule is that the arrows can't all point in the same direction. Use the template on the webpage to get started. Turn in a printout of your code and the plot.