Spring 2019/MA 262 **HW 1: A new view of old material**

Before beginning this homework assignment, please review the guidelines for submitting homework. In particular, please remember to turn in the cover sheet. **If you consult a classmate, online source, or textbook solutions** you must give credit for the help you received. Failure to do so may result in a report of academic dishonesty. You are, however, strongly encouraged to work with classmates – just be sure to give them credit for any ideas or help they provide!

If you are worried about how much time the assignment is taking you, please come see me. In general, you should find time spent on the course should be productive. Staring blankly at a piece of paper does not count as productive. Making mistakes, fixing them, reading the book, reviewing notes, rewriting your work more neatly, and talking with classmates is productive.

Problems marked with (CR) will be graded on a credit/no credit basis. Problems marked with (M) require you to use Mathematica. Any time you use Mathematica or other software to do a significant computation (such as an integral) you should say in your work that you have done so.

1. Getting started with Mathematica

This semester you will be required to use Mathematica to compute integrals and depict various functions. Part of your assignment this first week is to start learning it. Although it is extremely powerful, Mathematica is not always the most user friendly, so be mentally prepared to handle some frustration.

(1) Decide where you will use Mathematica. You can use it on any of the computers in Davis 216, or possibly elsewhere on campus. If you have your own computer, you can also install Mathematica free of charge for use while you are a Colby student. See the syllabus for instructions or ask for help.

(2) Watch the "Student's Introduction to Mathematica" available at http://www.wolfram.com/broadcast/video.php?c=89&v=269.

You can ignore the first minute of Mathematica propaganda and the last three minutes of extraneous advertising.

(3) From the course webpage, download the file "Mathematica Intro"; you can also follow this link. Open the file in Mathematica, read it, and use shift-return to evaluate the cells in order. Do they all work?

(4) Start a new Mathematica notebook. Save it using your last name as the file name. Do the following in this notebook, save it, print it (B& W is fine) and turn it in as part of your homework. You should take this opportunity to learn how to use Mathematica's documentation, so don't just get help from a friend - actually practice looking things up yourself. (But do get help if you need it, after trying it on your own.)

(a) Use Mathematica to calculate a numerical approximation to the value of this integral: $\int_0^\pi t\sin(t^5)\,dt$.

(b) Use Mathematica's documentation to learn how to find derivatives of functions. Find the third derivative of the the function $f(x) = x^3 e^{e^{3x}}$ using Mathematica.

(c) Plot the graph of the function $f(x) = x\sin\frac{1}{x^2}$ using a window whose x-axis ranges from $-\pi$ to 3π and whose y-axis ranges from -2 to 2.

2. Reviewing some Multivariable Calculus

These problems will probably feel a bit tedious. If so, that's good! It means you remember the things you're supposed to. Still I hope the review is helpful for resuscitating your calc skills.

(1) Read Sections 1.1 - 1.4 from the text. Except for the brief sections on parametric equations of lines, parametric equations in general, and matrices this should all be review from MA 122. (If you took MA 162, you might want to spend more time carefully reading this material.) **Email** me using the subject line "MA 262 HW 1" a list of topics you feel you don't understand and could use more review on.

(2) Suppose that A is the point in $\mathbb{R}^3$ with coordinates $(-5,1,7)$ and that the displacement vector from A to another point B is $(6,-4,2)$. What are the coordinates of B?

(3) Suppose that A is a point in $\mathbb{R}^2$ with coordinates (t,t^2) for some value of $t \in \mathbb{R}$ and that the displacement vector from A to a point B is $(\cos 2\pi t, \sin 2\pi t)$. What are the coordinates of B?

(4) Do problem 25, from page 8 of the text. Remember to explain your work and that you shouldn't consult the solutions manual for the text. If you do, be sure to cite it.

(5) Do problems 28, 29, 30, and 31 from the Exercises in Section 1.2. Remember to explain your work and that you shouldn't consult the solutions manual for the text. If you do, be sure to cite it.

(6) Use the dot product to find the projection of the vector $(1,5,20)$ onto the vector $(6,-4,-3)$.

(7) Use the cross product to find a vector in $\mathbb{R}^3$ perpendicular to the plane containing the vectors $(1,2,3)$ and $(3,-4,8)$. Do it by hand, but check your work using Mathematica.

(8) Find the determinants of the following matrices. Do it by hand, but check your work using Mathematica.

$$\begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 0 & 8 & 4 \\ 1 & 0 & 6 \end{pmatrix}$$

(9) Find all first partial derivatives of the following functions:
   (a) $f(x,y) = x^3 y - y^3 x$
   (b) $g(x,y) = \cos(x)\sin(y) - e^{3y}$
   (c) $h(x,y,z) = 5x^3 yz - \cos(yz) + \ln(x)$.

3. Matrix Mayhem

(1) Read Section 1.6. If you've taken Linear Algebra, this will be old stuff. But Linear Algebra isn't a prerequisite for this class, so if it's new to you, pay attention while you read!

(2) Calculate:

$$\begin{pmatrix} 1 & 5 \\ -2 & 6 \end{pmatrix} + \begin{pmatrix} 0 & 12 \\ 4 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 5 \\ -2 & 6 \end{pmatrix}\begin{pmatrix} 0 & 12 \\ 4 & -1 \end{pmatrix}, \begin{pmatrix} 1/3 & 2/3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

(3) Calculate and show your work (even though it is tedious and Mathematica will do it for you):

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 4 & 1 & -1 \\ 5 & -1 & 0 & 0 \\ -2 & 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ -3 & 1 & 0 & 1 \\ 6 & 1/2 & 4 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

(4) Do problems 14 and 15 from page 59 of your text. Remember to explain your work and that you shouldn't consult the solutions manual for the text. If you do, be sure to cite it.

4. DERIVATIVES

(1) Let $f: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by

$$f(x, y, z) = (3x^3 - xy, 2z \sin(xy)).$$

Find the derivative of f at the point $(1, 2, 0)$.

(2) Let $g: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$g(x, y) = (4e^x, x^2 + 2xy + y^2).$$

Compute $Dg(x, y)$ (the derivative of g at a generic point (x, y).)

(3) Use the chain rule to compute $D|_{(1,2,0)}(g \circ f)$ where f and g are the functions from the previous two problems.

(4) Find the affine approximation at $(1, 1, 1)$ to the function $h: \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$h(x, y, z) = (xy, -yz, xy - yz)$$

Also, is the function h orientation-preserving, orientation-reversing, or singular at $(1, 1, 1)$?

(5) Let A be an $n \times n$ matrix and let $\mathbf{b} \in \mathbb{R}^n$. Let $F: \mathbb{R}^n \to \mathbb{R}^n$ be the function defined by $F(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$. Explain why the derivative of F at any point is the matrix A.

5. COORDINATE CHANGES

(1) Recall that a function $f: \mathbb{R}^n \to \mathbb{R}^n$ (we just consider $n = 2, 3$) is said to be of class $\mathbf{C}^1$ if it is continuous and if all partial derivatives are continuous. Recall that f is **orientation preserving** at a point $\mathbf{a} \in \mathbb{R}^n$ if $\det Df(\mathbf{a})$ is positive. It is **orientation reversing** at $\mathbf{a}$, if $\det Df(\mathbf{a})$ is negative. It is **singular** at $\mathbf{a}$ if $\det Df(\mathbf{a}) = 0$.

Suppose that $f: \mathbb{R}^n \to \mathbb{R}^n$ is C^1 and is not singular at any point. Suppose also that f is orientation-preserving at some point $\mathbf{a}$. Explain why it must be orientation preserving at *all* points. Your answer should involve a careful reference to the intermediate value theorem from Calculus 1.

(2) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the change of coordinates transformation defined by $T(x, y) = (x^2 - y^2, x^2 + y^2)$.

(a) Determine the points where T is orientation-preserving, the points where T is orientation-reversing, and the points where it is singular.

(b) If we only allow ourselves to put values of (x, y) from the first quadrant into f, what possible points in $\mathbb{R}^2$ can we get out from f? (i.e. what is the range when the domain is the first quadrant?)

Hint: Show that T takes the positive y-axis to the ray of slope -1 based at the origin and above the x-axis. Also show that the positive x-axis gets taken to the ray of slope $+1$ based at the origin and above the x-axis.

(c) Consider the point $\mathbf{p} = (5, 1)$ in the first quadrant. Suppose that R is a very small region containing $\mathbf{p}$ and having area A. Approximate the area of the region $T(R)$ obtained by applying the transformation T to the region R. Also, speculate on how we might improve our approximation.

(3) Do problems 1, 4, 7, 10 from Section 1.7.

(4) Do problem 21 from Section 1.7. You are encouraged to try this by hand first and then to check it with Mathematica or other software.

(5) Do problem 26 from Section 1.7. You are encouraged to try this by hand first and then to check it with Mathematica or other software.