Spring 2018/MA 434 **HW 5: Cayley's Better Theorem**

1. Reading

Read Sections 1.1 - 1.5 from *Groups, Graphs, and Trees.* Answer the following reading comprehension questions:

(1) Definition 1.2 defines what is called a *right* action of a group on an object. What is a *left* action and how is it different from a right action? (Your answer may involve some speculation.) See Remark 1.4.

(2) In GP 1, you proved Cayley's Basic Theorem (Theorem 1.5). How is Meier's proof similar to yours? how is it different? which do you like better?

(3) What is the definition of a graph (Def. 1.9)? How does this differ from other definitions of graphs you may have seen?

(4) Come up with your own relatively complicated graph and specify a path of length at least 8 in it.

(5) If you haven't proved Exercise 1.10 in some other class, prove it now. (Hint: Induct on the number of edges in the tree.)

(6) **Claire** should be prepared to present Example 1.18 in class on Tuesday.

(7) **Teddy and Shabab** should be prepared to present solutions to Exercise 1.20 (a) and (b) in class on Tuesday.

(8) Memorize the definition of **free action** (Def. 1.29).

(9) **Brian** should be prepared to explain Example 1.40 in class on Tuesday.

(10) **Liwei** should be prepared to explain Example 1.41 in class on Tuesday.

2. Homework:

(1) Exercise 1.20 (c) - (f).

(2) The group $\mathbb{Z}^2$ (with $+$ as the operation) is generated by the elements $(1,0)$ and $(0,1)$. Draw (a large portion of) the Cayley graph for this generating set.

(3) The group $\mathbb{Z}^2$ (with $+$ as the operation) is generated by the elements $(1,1)$ and $(0,1)$. Draw (a large portion of) the Cayley graph for this generating set.

(4) Let C_n and C_m be cyclic groups of orders $n, m \geq 3$. Let t and s be generators for these groups. Prove that $\{t, s\}$ is a generating set for $C_n \times C_m$ and sketch the Cayley Graph. (Hint: think about what a torus looks like.)