Exam 1 Study Guide

On the exam you will only be asked to prove theorems you have previously proved - nothing new! Some of the group projects were based on material from Michael Artin's *Algebra*. That book is on reserve in the Olin science library.

(1) Know the definitions: group, left action, free group, orbit, stabilizer, transitive (action), free action, left action by translation, left action by conjugation, conjugacy class, the class formula, Cayley graph, fundamental domain, generating set for a group

(2) Be able to prove:

  (a) Orbit-Stabilizer Theorem (you may assume Lagrange's theorem)

  (b) Cayley's theorem: Every group is isomorphic to a subgroup of a permutation group

  (c) Cayley's better theorem: Every group is isomorphic to a subgroup of the symmetry group of some graph.

  (d) Orientation preserving and orientation reversing isometries of $\mathbb{R}^2$ are never conjugate.

  (e) Be able to prove the class formula for a finite group.

  (f) Be able to outline the proof that the icosahedral group is simple, using the class equation and symmetries of the dodecahedron.

  (g) Be able to outline the proof that the icosahedral group is isomorphic to A_5 (the alternating group on 5 elements).

  (h) The first and second Sylow theorems.

  (i) If a finite group has order divisible by a prime p, then the group has an element of order p.

  (j) $(\mathbb{Q},+)$ is not a finitely generated group.

  (k) If G acts on a connected graph G having each vertex of finite degree, then there is a fundamental domain for the action.

  (l) A free group on an alphabet $\mathscr{A}$ is a group.

  (m) Every element of a free group can be represented by a unique reduced word (the normal form theorem).

  (n) Theorem 1.55 (fundamental domains help us find generating sets)

  (o) Theorem 1.58 (if we know fundamental domains for the action of a group and a subgroup we can tell the index)

  (p) D_∞ has a subgroup of index 2 isomorphic to itself.

(3) Be able to construct Cayley graphs for the following groups:

  (a) finite and infinite dihedral groups

  (b) cyclic groups

  (c) $(\mathbb{Z},+)$

  (d) finite products of cyclic groups

  (e) free groups on a finite generating set