Spring 2018/MA 274 GP 1D: Modular Arithmetic

Consider the set of integers $\mathbb{Z}$.

(1) For integers x and y, define the symbol $x \equiv_5 y$ to mean that $x - y$ is a multiple of 5. More precisely, $x \equiv_5 y$ if and only if there is an integer $n \in \mathbb{Z}$ such that

$$x - y = 5n.$$

Explain why the following are true:

(a) For every $x \in \mathbb{Z}$, $x \equiv_5 x$. (To show this, you need to specify what value of n makes it so that $x - x = 5n$.)

(b) For every $x, y \in \mathbb{Z}$, if $x \equiv_5 y$, then also $y \equiv_5 x$. (You get to *assume* there is an n so that $x - y = 5n$. You need to *show* that $y \equiv_5 x$.)

(c) For every $x, y, z \in \mathbb{Z}$, if $x \equiv_5 y$ and $y \equiv_5 z$, then $x \equiv_5 z$.

(2) List all (or list some and say what the pattern is) the integers y such that $0 \equiv_5 y$.

(3) List all the integers y such that $1 \equiv_5 y$.

(4) List all the integers y such that $2 \equiv_5 y$.

(5) List all the integers y such that $3 \equiv_5 y$.

(6) List all the integers y such that $4 \equiv_5 y$.

(7) List all the integers y such that $5 \equiv_5 y$.

(8) Which of the sets you listed in (2) - (7) contains the number 1283? what about 1285?

(9) For an integer $p \in \mathbb{Z}$, define the notation $x \equiv_p y$ to mean that $x - y$ is a multiple of p. What has to change in your proofs from (1) if, in the statements, we change the number 5 to the number p?

(10) For an integer $x \in \mathbb{Z}$, let $[x]_p$ be the set of all integers y, such that $x \equiv_p y$. Show that if a, b, c are integers such that $c \in [a]_p$ and $c \in [b]_p$, then $a \equiv_p b$.

For your presentation, explain your answers to all these questions.