## Spring 2018/MA 274 GP 1C: Graphs

Read Section 2.3 up through Exercise 2.3.6.

(1) Do Exercise 2.3.5

(2) Figure 1 shows a way of moving through the graph along a path.

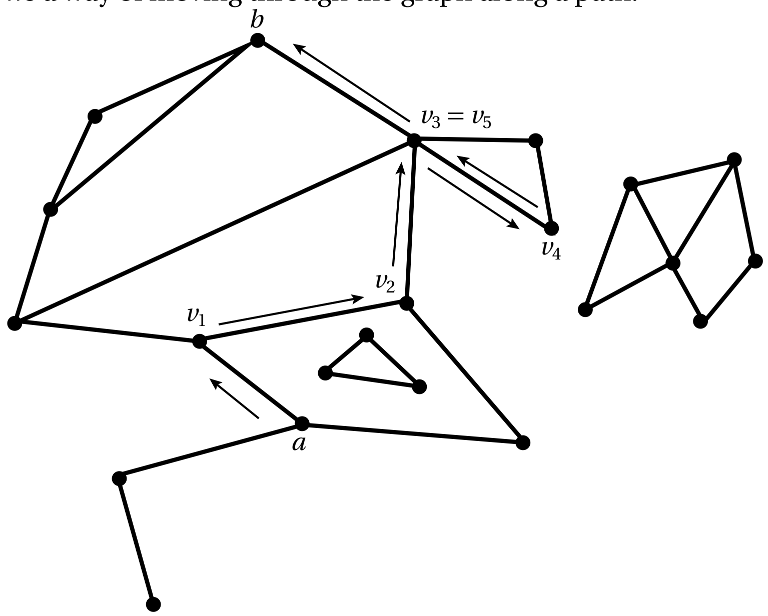


FIGURE 1. The finite sequence $a, v_1, v_2, v_3, v_4, v_5, b$ is a path from $a$ to $b$ of length 6.

To specify our route, we can list all the vertices we pass through:

$$a, v_1, v_2, v_3, v_4, v_5, b$$

The vertex $v_3$ is the same as the vertex $v_5$, so there are vertices that appear more than once in the list.

**Draw** your own example of a fairly complicated graph and specify a path in it between two vertices.

(3) Here is the formal definition of a path in a graph:

**Definition.** Suppose that $G = (V, E)$ is a graph . A **path** in $G$ is a finite sequence

$$\alpha = v_0, v_1, \dots, v_n$$

for some $n \in \mathbb{N}^*$ such that the following conditions hold:

- Each $v_i$ (for $i \in \{0, \dots, n\}$) is a vertex of $G$.
- For all $i \in \{0, \dots, n-1\}$, the vertices $v_i$ and $v_{i+1}$ are the endpoints of an edge in $G$.

The number $n$ is the **length** of $\alpha$. If $e$ is an edge of $G$ such that there exists $i \in \{0, \dots, n-1\}$ with the endpoints of $e$ equal to $v_i$ and $v_{i+1}$, then we say that $e$ is **traversed** by $\alpha$. If $a$ and $b$ are vertices of $G$ such that $v_0 = a$ and $v_n = b$, we say that $\alpha$ is a path **from** $a$ **to** $b$.

**Explain** how the formal definition applies to the example given above and to the example you created.

(4) Suppose that $G$ is a graph. For vertices $a$ and $b$ define the symbol $a \sim b$ to mean that there is a path from $a$ to $b$. Explain why the following are true:

   (a) For every vertex $a$, we have $a \sim a$. (Hint: A single vertex can consist of a path.)
   (b) For every pair of vertices, if $a \sim b$ then also $b \sim a$. (Hint: Assume that there is a path from $a$ to $b$, explain why there must be a path from $b$ to $a$.)
   (c) For every three vertices $a$, $b$, and $c$, if $a \sim b$ and $b \sim c$, then $a \sim c$.

(5) Explain what it means if for every two vertices $a$ and $b$ in the graph $G$ we have $a \sim b$.

(6) Give an example of a graph such that there are vertices $a$ and $b$ such that $a \not\sim b$.

For your presentation, explain the definition of path in a graph and your answers to the last three problems.