Spring 2018/MA 274 GP 1A: Graphs

Read Section 2.3 up through Exercise 2.3.6.

(1) Do Exercise 2.3.5

(2) What matters in a graph is what the connections are, not how we draw it. In the figure below are two pictures of a particular graph called the Petersen graph. Label the vertices in each diagram and explain how to match the vertices in the left diagram with the vertices in the right diagram so that two vertices on the left are joined by an edge if and only if the corresponding vertices on the right are joined by an edge.

(3) Consider the set $A = \{1,2,3,4,5\}$. Create a graph as follows. For each two element subset (for instance $\{1,4\}$) create a vertex. Join two vertices together by an edge if the associated subsets have no elements in common. Draw a picture of this graph. Label each vertex with the corresponding subset.

(4) Show that the graph in the previous problem is the same graph as the Petersen graph. You can do this by explaining how to match the vertices in your drawing with the vertices in one of the drawings above.

(5) Suppose that G_1 and G_2 are graphs. We say that G_1 is **isomorphic** to G_2 if there is a way of matching the vertices of G_1 with the vertices of G_2 so that every vertex of G_1 is matched with a vertex of G_2 and vice versa and two vertices of G_1 are joined by an edge if and only if the corresponding vertices of G_2 are joined by an edge.

Explain why each of the following is true, by using the definition of isomorphic.

(a) Every graph is isomorphic to itself.
(b) If G_1 is isomorphic to G_2, then G_2 is isomorphic to G_1.
(c) If G_1 is isomorphic to G_2 and if G_2 is isomorphic to G_3, then G_1 is isomorphic to G_3.

In your presentation, you'll say what a graph is, and present your answers to 3, 4, 5.