S16 MA 274: Group Project 2

You will work in teams of 2 or 3 to complete this project. Each person in the group must turn in (on Monday **February 22**) the following **typed** in LaTeX. You may hand-draw any pictures that you need. The proofs must be found through group discussion, but the write-ups should be done independently. You should not consult any sources other than your course notes or texts.

(1) The statement and proof of the theorem below. (You should do but not turn-in the warm-up exercises.)
(2) A paragraph naming each person in the team, and summarizing the process by which the team came to find the proofs. You should address such questions as: How difficult was it to figure out what to prove? Did the team talk through everything together or did they work independently and then compare solutions? Did one person (who?) take the lead or provide all the ideas? Did you feel comfortable questioning your teammates solutions? why or why not? What did you contribute to the process?

The project:

The word “graph” in mathematics has several meanings. One of the meanings is that a graph G consists of a set of vertices (or nodes) and a set of edges (or connections). We’ll call the set of vertices $V(G)$ and the set of edges $E(G)$. If $v,w \in V(G)$ are distinct then the set $\{v,w\}$ is called the **edge** between v and w. Every element of $E(G)$ is a set of the form $\{v,w\}$ where v and w are distinct elements of $V(G)$.

If $e = \{v,w\} \in E(G)$, we can form a new graph denoted $G-e$ by letting $V(G-e) = V(G)$ and $E(G-e) = \{f \in E(G) : f \neq e\}$. The graph $G-e$ is said to be obtained by **removing** the edge e from $E(G)$.

A **path** in a graph G is a list of vertices

$$v_0, v_1, v_2, \ldots, v_n$$

(for some $n \in \mathbb{N} \cup \{0\}$) such that for each $i \in \{0,\ldots,n-1\}$, the set $\{v_i, v_{i+1}\} \in E(G)$. (That is, adjacent vertices in the path are endpoints of an edge in G.) We say that the path is **from** v_0 to v_n. The path is a **cycle** if $v_0 = v_n$ and if for all $i \in \{1,\ldots,n-1\}$, $v_{i-1} \neq v_{i+1}$ (i.e. no “back-tracking”).

Warm-up Exercise 1: Draw some examples of graphs and paths in graphs.

A graph is **connected** if whenever $v,w \in V(G)$, there exists a path from v to w. If G is connected and if $e \in E(G)$ but $G-e$ is disconnected, then we say that e **disconnects** G.

Warm-up Exercise 2: Draw some examples of connected graphs and some examples of disconnected graphs.

Theorem. Suppose that G is a connected graph. Then every edge of G disconnects G if and only if G does not have a cycle.

Group Assignments

- Mirco Dinelli, Sarah Whitney, Fan Zhang
- Chowdhury Farabee, Abby Johnson, Katherine Zafirson
- Keith Barnatchez, Hannah Bossi, Katie Discipio
- Jason Beland, Luc Marrie, Pearson Treanor
- John Baron, Grace Farnkoff, Stacey Hou
- Robert Campbell, Tomo Cho, Benard Kibet
- Eric Hartman, Makoto Kinoshita, Xuning Wang
- Avery Heilbron, Joshua Young
- Hanjing Shi, Zitong Wang