MA 274: Exam 2 Study Guide

Here are some suggestions for what and how to study:

(1) Know the definitions on the website. Any other definitions that you need will be given to you.

(2) When you write a proof, focus on getting the organization clear and correct. If you have to skip some steps or make an assumption that you don't know how to prove, clearly state that that is what you are doing.

(3) Know the theorems we've proved in class and the more significant theorems from the homework.

(4) Don't try to memorize proofs. Instead remember the structure of the proof (proof by contradiction, proof of uniqueness, element argument, etc.) and two or three key steps of the proof. Then at the exam recreate the proof.

(5) At the exam, leave time to write up a nicely written version of each proof. You should have enough time to sketch your ideas out on scratch paper before writing a final version of the proof.

(6) Here are some results you should be especially sure to know how to prove. You should also think about ways these problems might be varied. And you should study other problems too.

  (a) The compositions of injections/surjections/bijections is a an injection/surjection/bijection.

  (b) A function $f\colon X \to Y$ is a bijection if and only if there is a function $f^{-1}\colon X \to Y$ such that $f \circ f^{-1}(y) = y$ for all $y \in Y$ and $f^{-1} \circ f(x) = x$ for all $x \in X$.

  (c) Basic proofs by induction (see text, homework, and class notes)

  (d) If X is a set with n elements, then every permutation (i.e. bijection $X \to X$) is the composition of transpositions. (Theorem 6.1.7 - we did this in class)

  (e) Euler's theorem for planar graphs: If G is a finite, planar, non-empty, connected graph with $V(G)$ vertices, $E(G)$ edges, and $F(G)$ faces, then $V(G) - E(G) + F(G) = 2$.

  (f) The Well-Ordering Principle

  (g) Every integer greater than one is a multiple of a prime number.

  (h) Every fraction can be written in lowest terms.

  (i) $\operatorname{card} \mathbb{N} \times \mathbb{N} = \operatorname{card} \mathbb{N}$ (the Cantor Snake)

(j) The rationals are countable

(k) The set of algebraic numbers is countable

(l) The countable union of countable sets is countable.

(m) The interval $(0,1)$ and the set $\mathbb{R}$ are uncountable

(n) For every set X, $\operatorname{card} X < \operatorname{card} \mathscr{P}(X)$.

(o) $\operatorname{card} \mathbb{R} = \operatorname{card} \mathscr{P}(\mathbb{N})$.

(p) Let $\mathscr{F}$ be the set of functions from $X \to \{0,1\}$. Then $\operatorname{card}(\mathscr{P}(X)) = \operatorname{card} \mathscr{F}$.

(q) The proofs from Group Project 3 (I guarantee that one of these will be on the exam)

(r) If Y is a set and if $g\colon\ \mathbb{N} \to Y$ is a function such that for all $N \in \mathbb{N}$,

$$\operatorname{range} g \neq \{g(1), \ldots, g(N)\}$$

then there exists an injective function $f\colon\ \mathbb{N} \to Y$ such that $\operatorname{range} f = \operatorname{range} g$.

(s) A finite connected graph contains an euler cycle (i.e. a cycle traversing every edge exactly once) if and only if every vertex has even degree. (The Königsberg bridge problem)

(t) A set $A \subset \mathbb{R}$ with the property that $\sup A \in \mathbb{R}$ contains an increasing sequence.

(u) Theorem 8.3.16: An increasing sequence in $\mathbb{R}$ is either eventually constant or has a subsequence which is strictly increasing.

(v) Every sequence in $\mathbb{R}$ has a monotonic subsequence.

(7) Here are a few problems you haven't done before:

(a) If X is countable and if $A \subset X$, then A is countable.
(b) If there are bijections $X \to \{1, \ldots, n\}$ and $X \to \{1, \ldots, m\}$ for some $n, m \in \mathbb{N}$. Then $n = m$. (Be careful: this is harder than it looks!)