MA 274: Exam 1 Study Guide

Here are some suggestions for what and how to study:

(1) Know the definitions on the website from Chapters 1 - 4. Any other definitions/axioms that you need will be given to you.

(2) When you write a proof, focus on getting the organization clear and correct. If you have to skip some steps or make an assumption that you don't know how to prove, clearly state that that is what you are doing.

(3) Know the theorems we've proved in class and the more significant theorems from the homework.

(4) Be able to correctly write the negation of a statement.

(5) Don't try to memorize proofs. Instead remember the structure of the proof (proof by contradiction, proof of uniqueness, element argument, etc.) and two or three key steps of the proof. Then at the exam recreate the proof.

(6) At the exam, leave time to write up a nicely written version of each proof. You should have enough time to sketch your ideas out on scratch paper before writing a final version of the proof.

(7) Here are some results you should be especially sure to know how to prove. You should also think about ways these problems might be varied. And you should study other problems too.

   (a) The number $\sqrt{2}$ is irrational.
   (b) There are infinitely many prime numbers.
   (c) There is no set U such that $A \in U$ if and only if A is a set. (Russell's Paradox)
   (d) The Halting Problem
   (e) DeMorgan's Laws
   (f) A group has a unique identity.
   (g) The intersection of subgroups is a subgroup
   (h) The intersection of convex sets is convex
   (i) The intersection of event spaces is an event space.
   (j) If X is a metric space and if U and V are open subsets of X, then $U \cap V$ is open.
   (k) $X \times Y = Y \times X$ if and only if either $X = Y$ or one of X or Y is empty.

(8) Here is some new ones for you to prove:

   (a) Suppose that X is a metric space and that $x \in X$ and $r > 0$. Then $B_r(x)$ is open. (Hint. Let $y \in B_r(x)$. You must show that there is $s > 0$ such that $B_s(y) \subset B_r(x)$. In other words, find s (which may depend on x, y, and r) so that if z is within distance s of y then it is also within distance r of x.)

(b) Suppose that $\mathscr{E}$ is an event space and that for each $i \in \mathbb{N}$, V_i is an event. Prove that $\bigcap_{i=1}^{n} V_i$ is an event.

(c) Suppose that G is a graph and that for each $\lambda \in \Lambda$, H_λ is a **subgraph** of G. That is, H_λ is a graph (so the endpoints of each edge in H_λ are vertices of H_λ) and $V(H_\lambda) \subset V(G)$ and $E(H_\lambda) \subset E(G)$. Prove that $\bigcap\limits_{\lambda \in \Lambda} H_\lambda$ is a subgraph of G. To do that you must show:

- Every vertex of $\bigcap\limits_{\lambda \in \Lambda} H_\lambda$ is also a vertex of G (should be easy)
- Every edge of $\bigcap\limits_{\lambda \in \Lambda} H_\lambda$ is also an edge of G (should be easy)
- The endpoints of every edge in $\bigcap\limits_{\lambda \in \Lambda} H_\lambda$ are vertices in $\bigcap\limits_{\lambda \in \Lambda} H_\lambda$ (might be harder)