MA 274: Convex Sets

Definition. A subset $U \subset \mathbb{R}^2$ is **convex** if for all $P,Q \in U$ the line segement $\overline{PQ}$ between P and Q is also a subset of U.

Theorem 0.1. Suppose that $\mathscr{C}$ is a set whose elements are convex subsets of $\mathbb{R}^2$. Then $\bigcap \mathscr{C}$ is convex.

Proof. Let $P,Q \in \bigcap \mathscr{C}$. We must show that the line segment $\overline{PQ}$ is a subset of U. Since both $P,Q \in \bigcap \mathscr{C}$, by the definition of "intersection", for every convex set $C \in \mathscr{C}$, we have $P,Q \in C$. Since each $C \in \mathscr{C}$ is convex, by the definition of convex, the line segment $\overline{PQ} \subset C$. Let $x \in \overline{PQ}$. By the definition of subset, for every $C \in \mathscr{C}$, the point $x \in C$. Thus, by definition of intersection, $x \in \bigcap \mathscr{C}$. Since this is true for every $x \in \overline{PQ}$, we have $\overline{PQ} \subset \bigcap \mathscr{C}$, as desired. Thus, $\mathscr{C}$ is convex. $\square$