Exam 1 Study Guide

MA 111 Spring 2015

1. Reading Questions

It is suggested you review the reading and the examples of different kinds of symmetries from the reading.

(1) Be able to name all the types of symmetries of a given pattern, using proper terminology

(2) Be able to recognize frieze patterns and wallpaper patterns

(3) Be able to explain the different kinds of symmetries which may or may not be present in a musical canon.

(4) Be able to draw shapes exhibiting certain kinds of symmetries.

2. Some math

(1) Give an example of a planar shape with non-trivial rotational symmetry.

(2) Give an example of a planar shape with non-trivial rotational symmetry, but no bilateral symmetry.

(3) Define the following groups or terminology:

translation	reflection
glide reflection	
group	even permutation
D_n	subgroup
$\mathbb{S}_n$	cycle number
transposition	odd permutation
C_n	generators for a group
braid group	identity in a group
associative property	inverse of a symmetry
generators	$\mathbb{A}_n$
frieze pattern	wallpaper pattern

(4) Is $\mathbb{R}$ (the real numbers) with addition a group? Is $\mathbb{R}$ with subtraction a group?

(5) Carefully explain why D_n can be generated by a rotation and a reflection.

(6) Carefully explain why D_n can be generated by two reflections.

(7) Carefully explain why $\mathbb{S}_n$ can be generated by transpositions.

(8) Carefully explain why each symmetry in $\mathbb{S}_n$ is either a combination of an even number of transpositions or an odd number of transpositions, but not both.

(9) Carefully explain why the 14-15 puzzle cannot be solved.

(10) Be able to determine if a given sliding block puzzle cannot be solved using the method from class.

(11) Explain the relationship between plain bob minimus (which would be given to you, no need to memorize), the subgroup $\langle [1\,2][3\,4], [2\,3] \rangle$ of $\mathbb{S}_4$, and the cosets of that subgroup.

(12) Find all symmetries in the subgroup $\langle [1\,2\,3]\,[4\,5]\,, [4\,5] \rangle$ of $\mathbb{S}_5$.

(13) Show that R_{144} generates C_5

(14) How many symmetries are in the subgroup $\langle [1\,2\,3]\,[4\,5] \rangle$ of $\mathbb{S}_5$?

(15) How many symmetries are in the subgroup $\langle [1\,2\,3]\,[4\,5] \rangle$ of $\mathbb{S}_5$?

(16) Suppose that S and T are two symmetries in a group. Show that $(T \circ S)^{-1} = S^{-1} \circ T^{-1}$.

(17) Explain why D_n is a subgroup of $\mathbb{S}_n$ for all n.

(18) Given a subgroup of a group be able to find all the subgroups of that group.

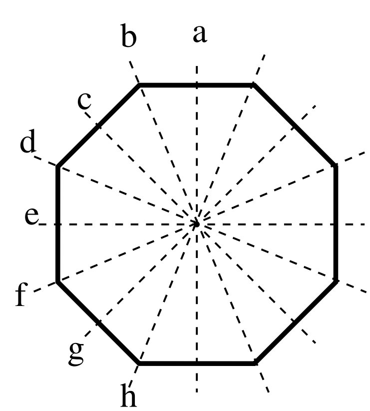
(19) Be able to use LaGrange's theorem to find the the number of cosets of a given group

(20) Be able to explain how LaGrange's theorem give restrictions on the symmetry groups of decorated objects.

(21) The subgroup D_8 has 8 rotations and 8 reflections. The rotations are

$$\mathbf{I}, R_{45}, R_{90}, R_{135}, R_{180}, R_{225}, R_{270}, R_{315}.$$

The reflections are drawn below.



Perform the following computations

(a) $R_{45} \circ a$

(b) $a \circ R_{45}$

(c) $R_{135} \circ g$

(d) $R_{135} \circ g \circ R_{135}$

(22) Perform the following computations in $\mathbb{S}_5$:

(a) $[1\,2\,3]\,[2\,5\,4]$

(b) $[1\,2]\,[2\,3]\,[3\,5]\,[5\,4]$

(23) Let $g = [1\,2\,3\,4\,5\,6][7\,8\,9]$ in $\mathbb{S}_9$.

(a) Let $t = [2\,5]$. Draw convincing pictures to show how the cycle number of $g \circ t$ differs from the cycle number of g.

(b) Let $t = [4\,8]$. Draw convincing pictures to show how the cycle number of $g \circ t$ differs from the cycle number of g.

(24) Determine whether the following symmetries are odd or even permutations.

(a) $[2\,4\,6][1\,3]$

(b) $[1\,2\,3\,4][5\,6\,7]$

(c) $[2\,3\,9][1\,4\,8\,5][10\,11\,5]$

(d) $[2\,3\,4\,5][3\,4\,5\,6]$.

3. Cosets and LaGrange's Theorem

(1) State LaGrange's Theorem and explain, in detail, why it is true.

(2) Use LaGrange's theorem to show that if a group contains a prime number of symmetries then the only subgroups are $\{\mathbf{I}\}$ and the whole group. Use this to show that any symmetry other than $\mathbf{I}$ will generate the group.

(3) Consider the subgroup

$$\langle [1\,2\,3\,4], [3\,4\,5], [4\,5\,6\,7\,8] \rangle$$

of $\mathbb{S}_8$. Explain why this subgroup must contain at least 60 symmetries.

(4) Refer to the notation for symmetries in D_8 above. Let H be the subgroup $\{\mathbf{I}, R_{90}, R_{180}, R_{270}\}$. Organize the symmetries in D_8 according to the cosets of H in D_8.