MA 314: Exam 2

The Rules: You have from the time you receive the exam until 5 PM on Thursday, April 17 to complete the exam and turn in your answer. You may use your textbooks and class notes, but you may not use any other resources in print or online. You may not communicate about the exam with anyone except the professor.

Sign here after completing the exam to indicate that you have read and abided by the rules:

A word of advice: Start this early. Don't turn in your first drafts. Turn in beautiful, readable, clear work.

Do three of the following 4 problems.

(1) In class we showed how to produce a regular octogon $X \subset \mathbb{H}^2$ with internal angle sum 2π. The standard gluings produce an genus 2 surface $\overline{X}$. The edges of X become closed geodesics in $\overline{X}$ all of the same length λ.

  (a) Prove that if $g \subset \overline{X}$ is a closed geodesic then the length of g is not **smaller** than λ.

  (b) Show how to produce an irregular octogon $Y \subset \mathbb{H}^2$ with internal angle sum 2π such that there are isometries between pairs of edges so that the glued up surface $\overline{Y}$ is also a genus 2 surface. Do this so that $\overline{Y}$ is not isometric to $\overline{X}$. Explain how you know that $\overline{Y}$ is not isometric to $\overline{X}$.

(2) Use the tiling theorems discussed in class to prove that if a triangle $T \subset \mathbb{E}^2$ tiles the plane by reflecting over the sides of T, then the angles of T are of the form $\frac{2\pi}{k}$, $\frac{2\pi}{\ell}$, and $\frac{2\pi}{m}$ for some natural numbers k, l, m. Determine exactly which angles will produce a triangle which can tile $\mathbb{E}^2$. Then do the same thing for triangles in the sphere S^2.

(3) Let T^{**} be the twice punctured torus (i.e. the torus minus 2 points). Produce a polygon $X \subset \mathbb{H}^2$ and gluing maps so that $\overline{X}$ is homeomorphic to T^{**}. (Note that the polygon will necessarily have ideal vertices.)

(4) Consider a connected polygon $X \subset \mathbb{H}^2$ with gluing maps so that we get a tiling of $\mathbb{H}^2$ as in Bonahon Theorem 6.1. For a point $x \in \mathbb{H}^2$, pick a tile X' containing x and let $\pi(x) \in \overline{X}$ be the corresponding point in the glued up surface. Observe that if ϕ is the element from the tiling group so that $\phi(X) = X'$ we can define $\pi(x) = \overline{\phi^{-1}(x)}$. This does not depend on the tile X' containing x. We now have a continuous function $\pi\colon \mathbb{H}^2 \to \overline{X}$. Prove (by appealing to facts from class/Bonahon and some additional work, as necessary) that a curve $\overline{g} \subset \overline{X}$ is a geodesic if and only if there is a hyperbolic geodesic $\widetilde{g} \subset \mathbb{H}^2$ with

$$\pi(\widetilde{g}) = g$$