TREES, POLYGONS, HYPERBOLIC GEOMETRY

2	3	5	7	11	13	17	19	23	29
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877	881	883	887	907	911	919	929	937	941
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1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
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1993	1997	1999	2003	2011	2017	2027	2029	2039	2053
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2293	2297	2309	2311	2333	2339	2341	2347	2351	2357
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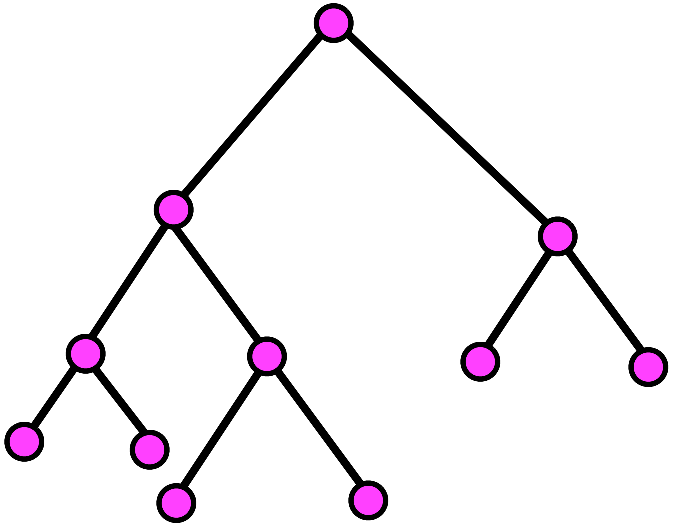
TREES



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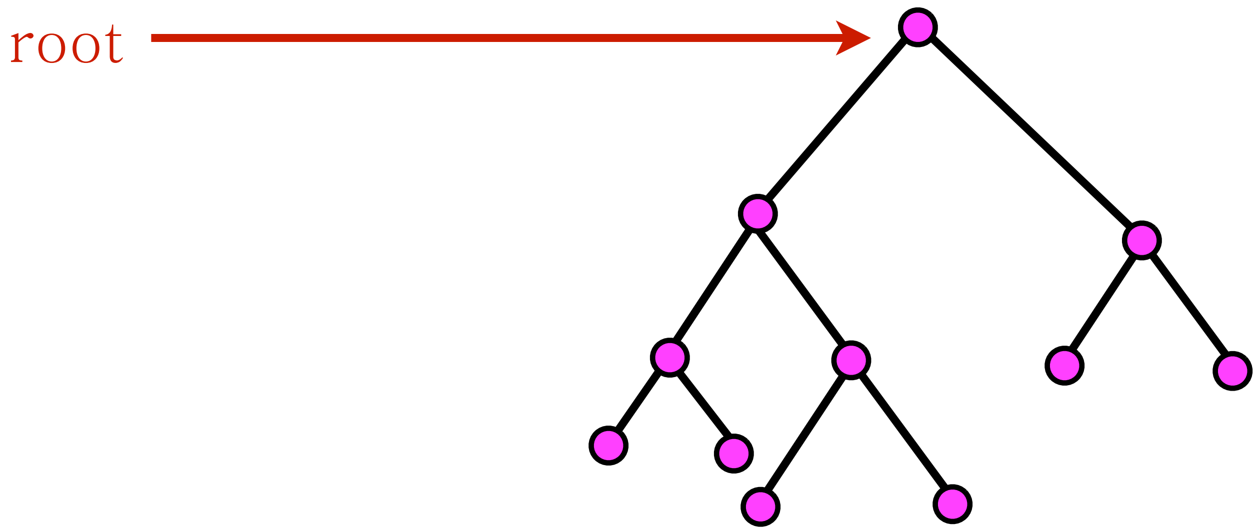

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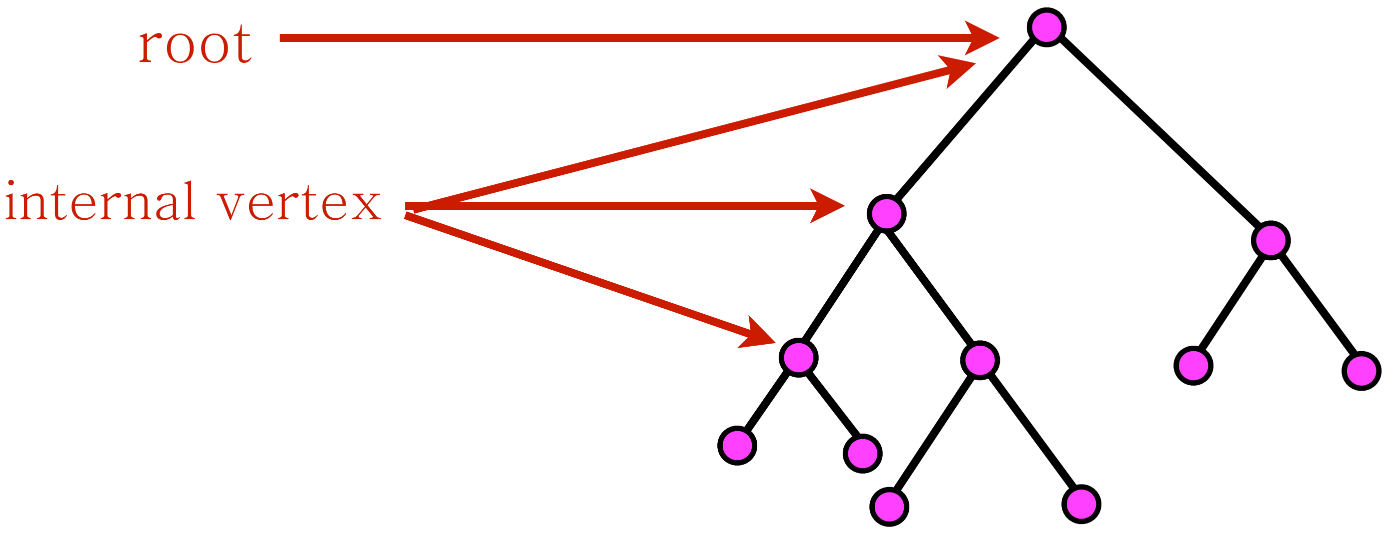

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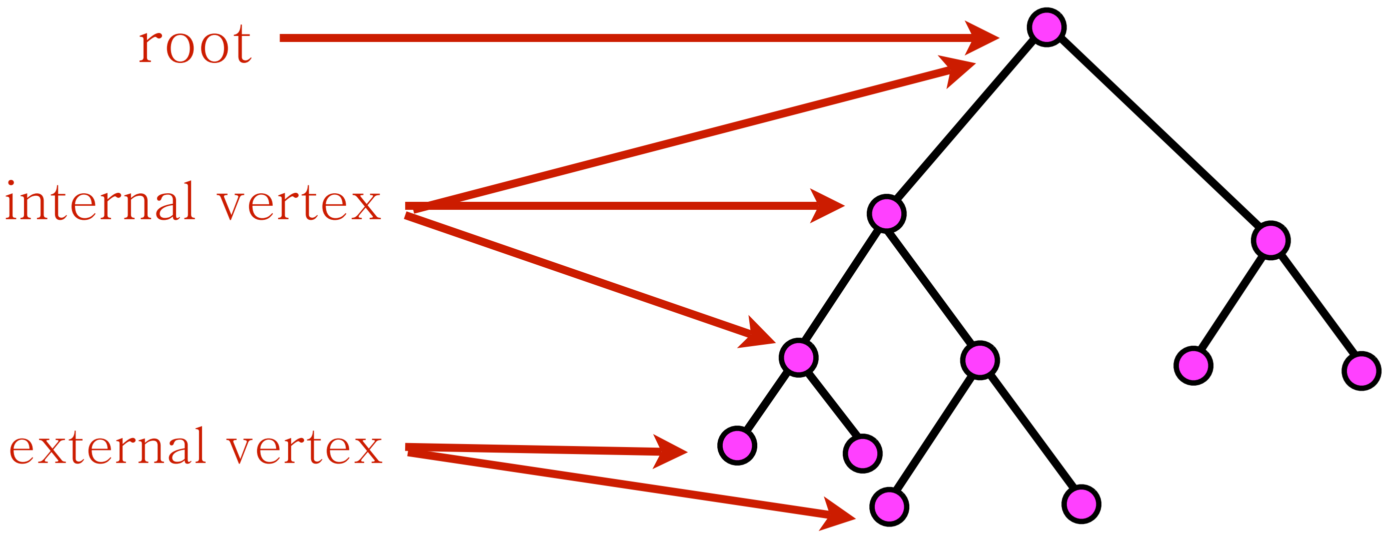

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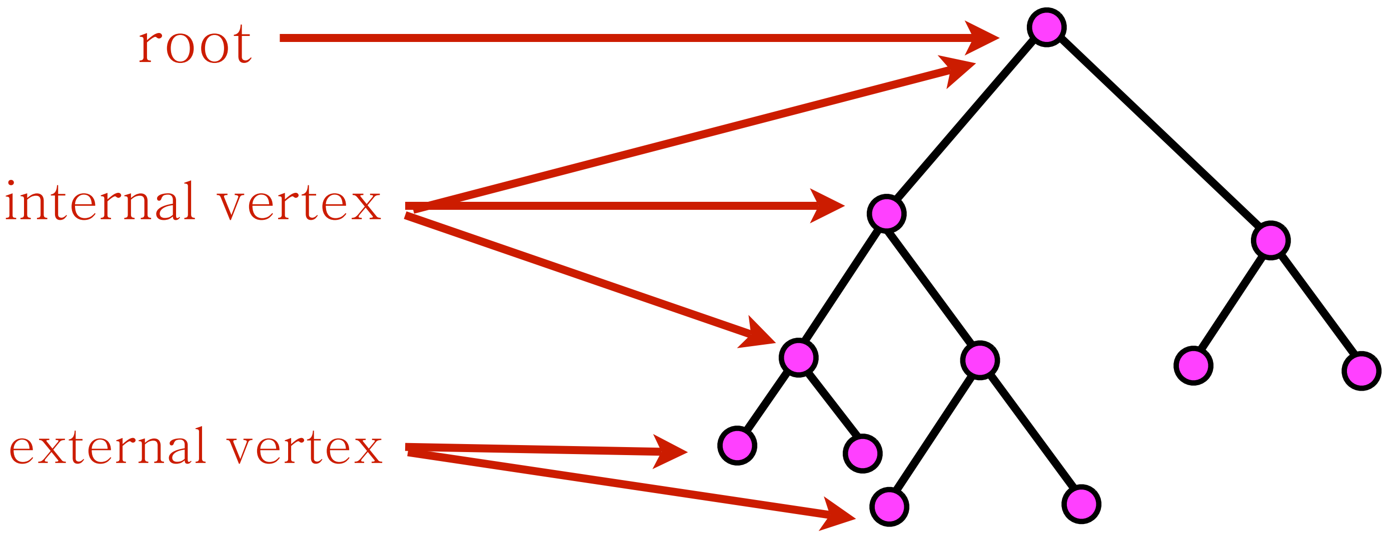
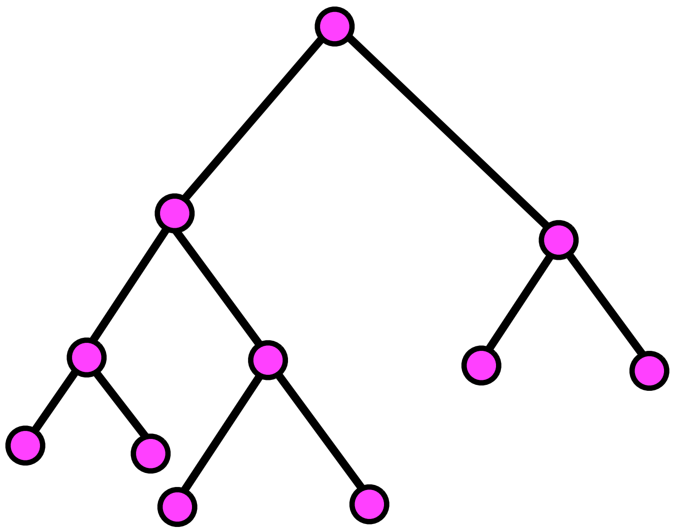
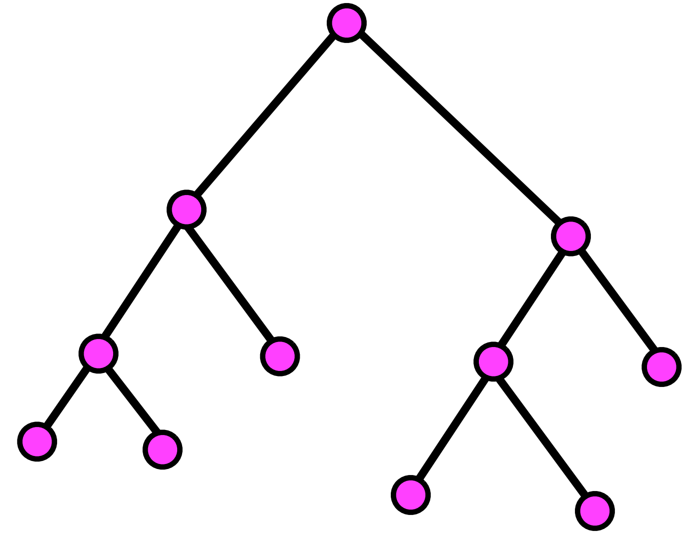


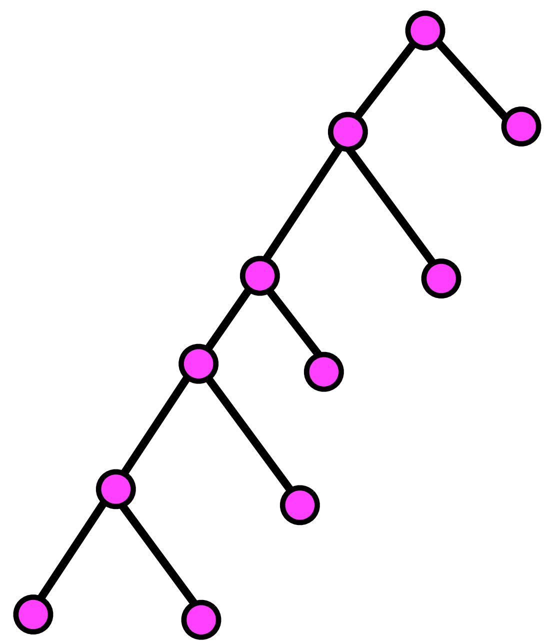
size = # of internal vertices

TREES





symmetric
numbering of
internal vertices



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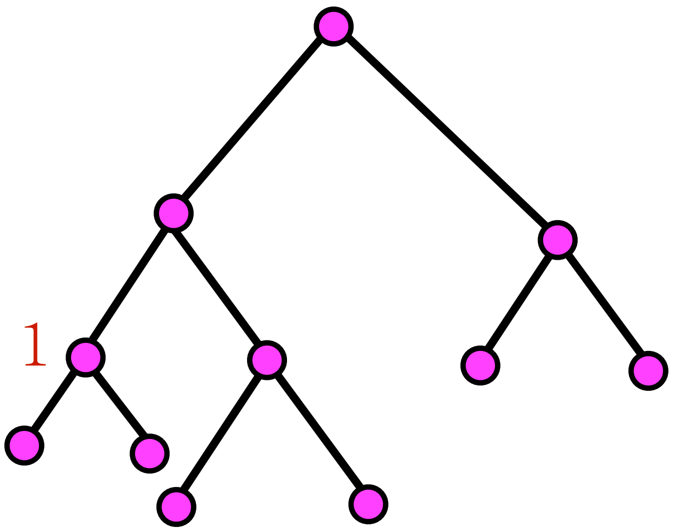
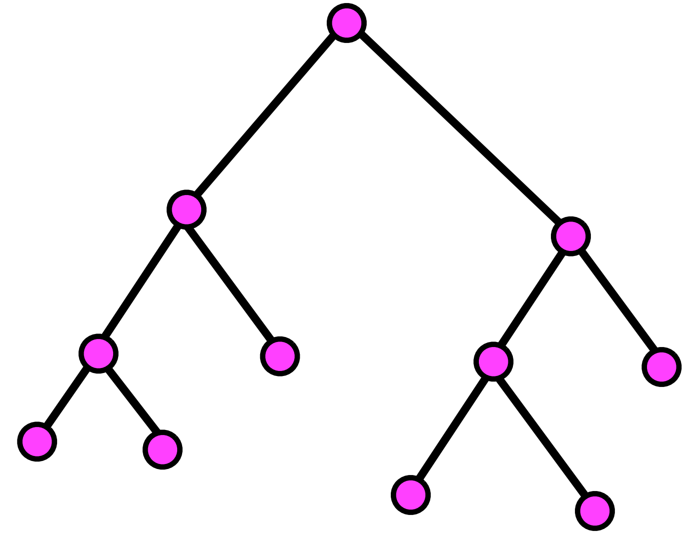
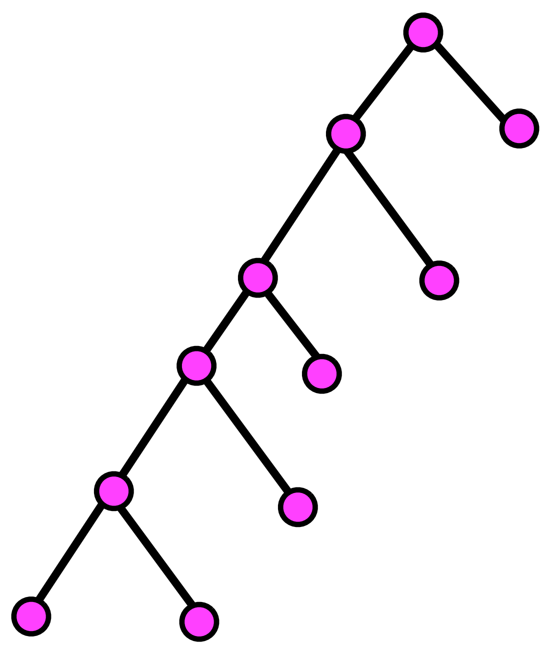
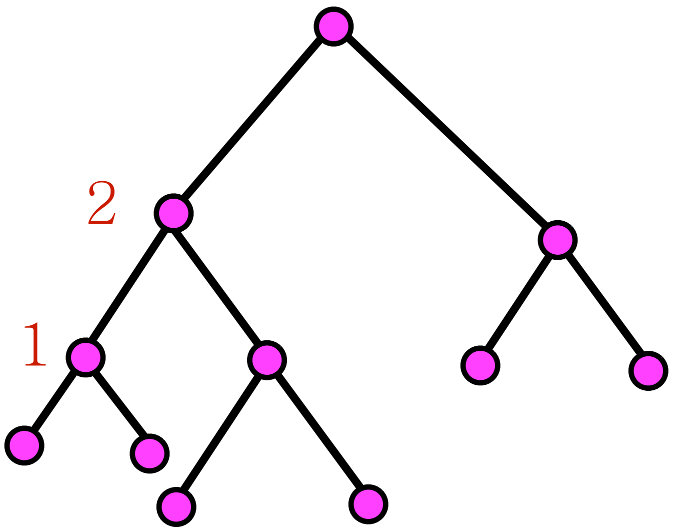


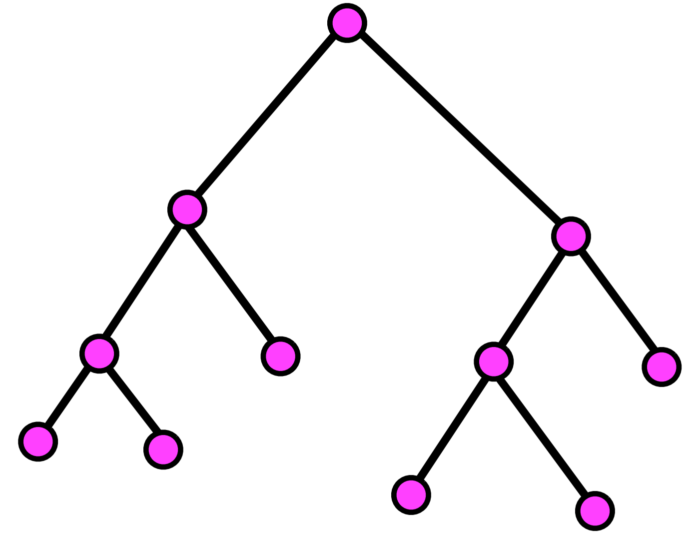


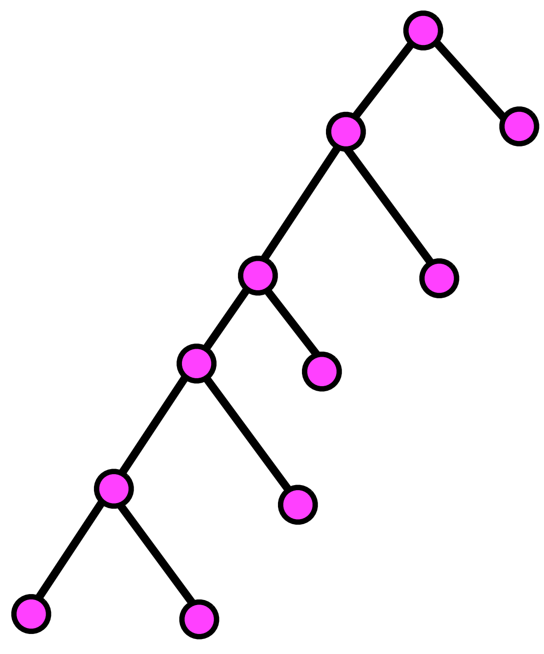


symmetric
numbering of
internal vertices

TREES

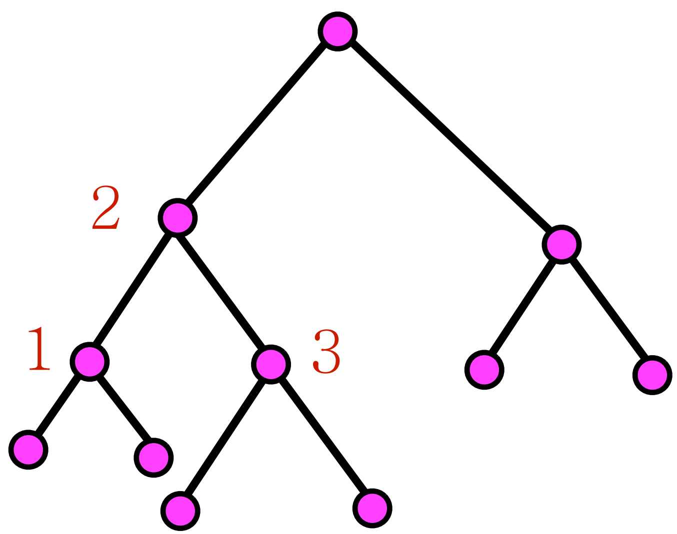


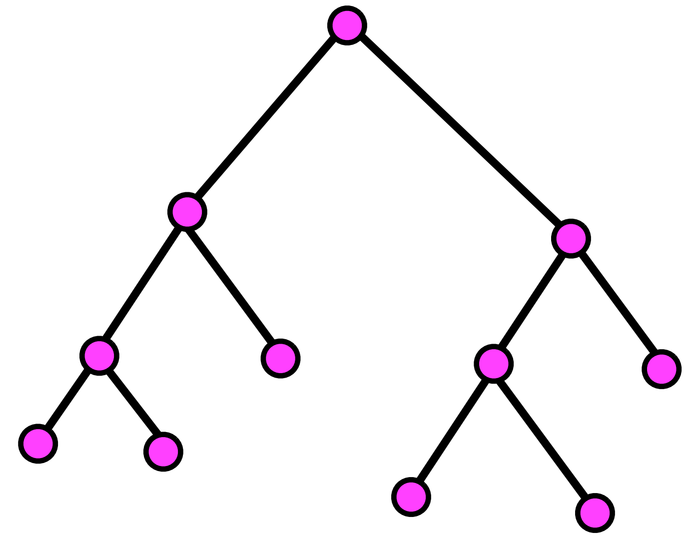


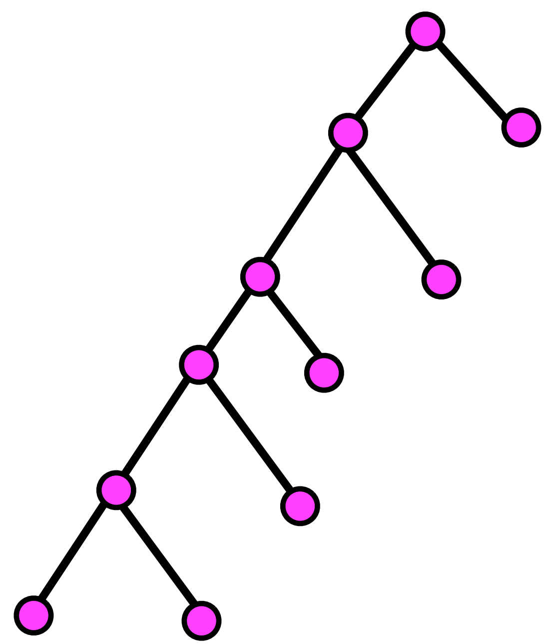


symmetric
numbering of
internal vertices

TREES

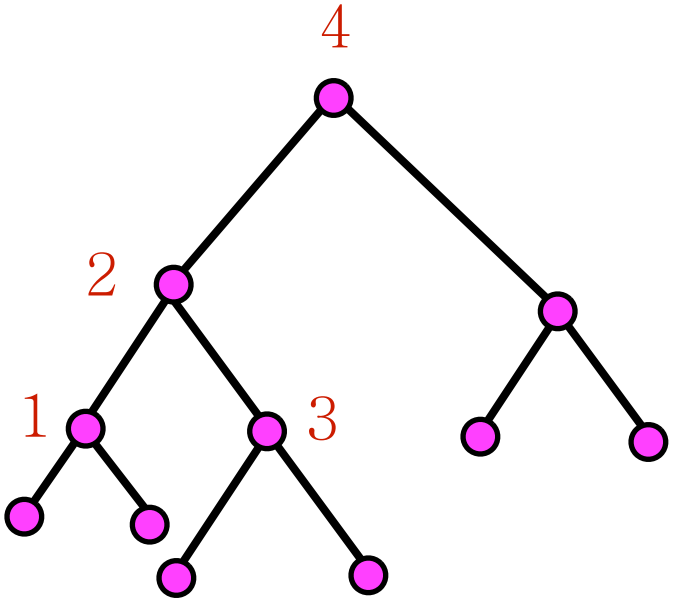


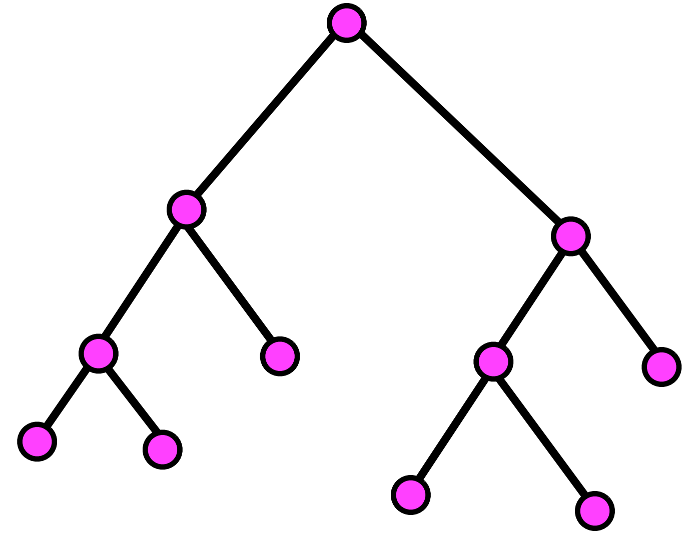


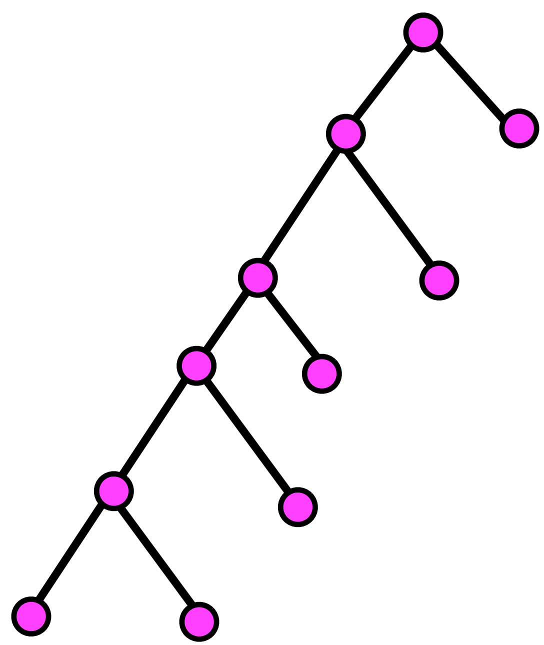


symmetric
numbering of
internal vertices

TREES

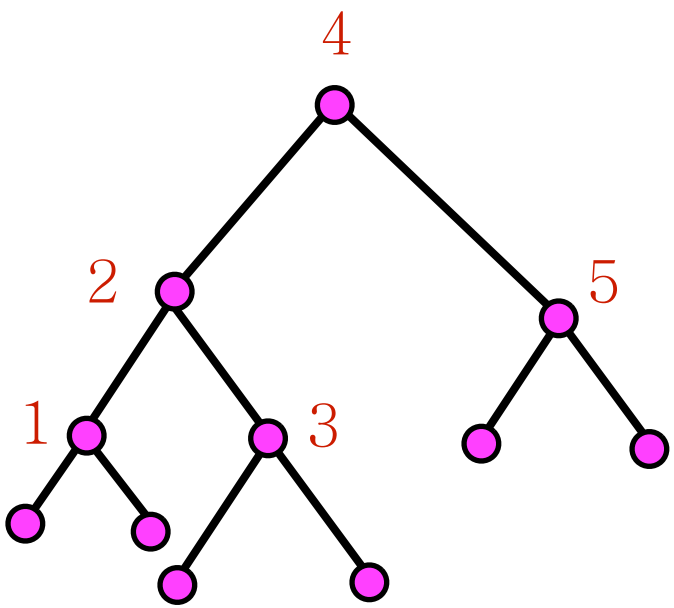


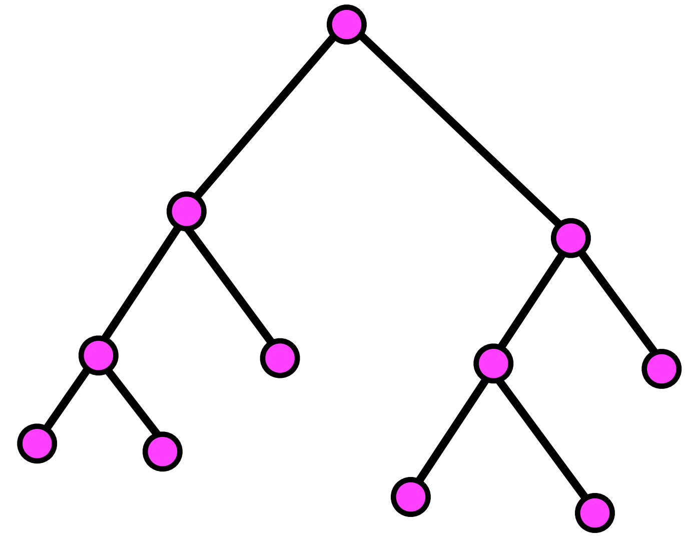


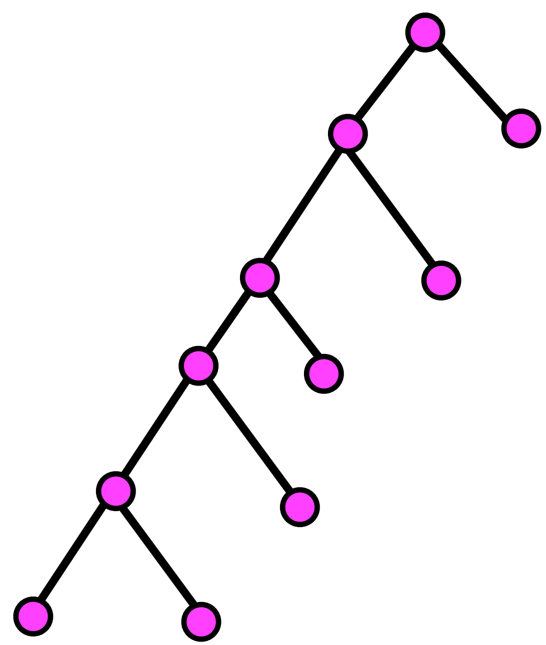


symmetric
numbering of
internal vertices

TREES

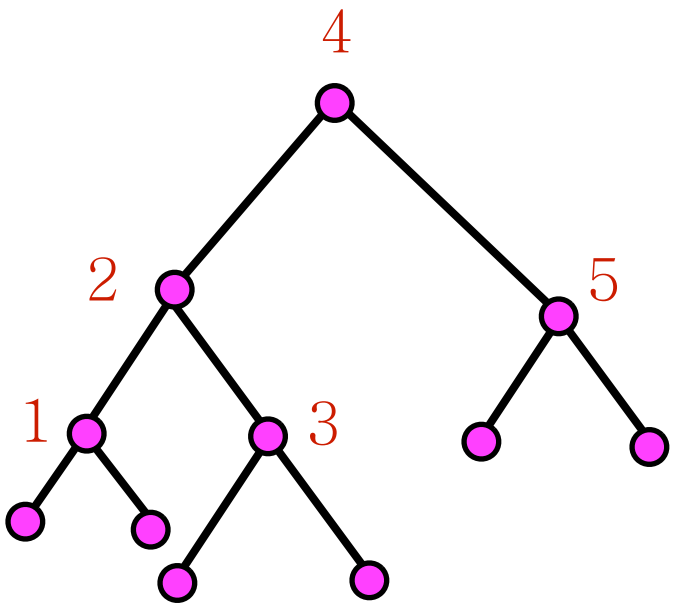


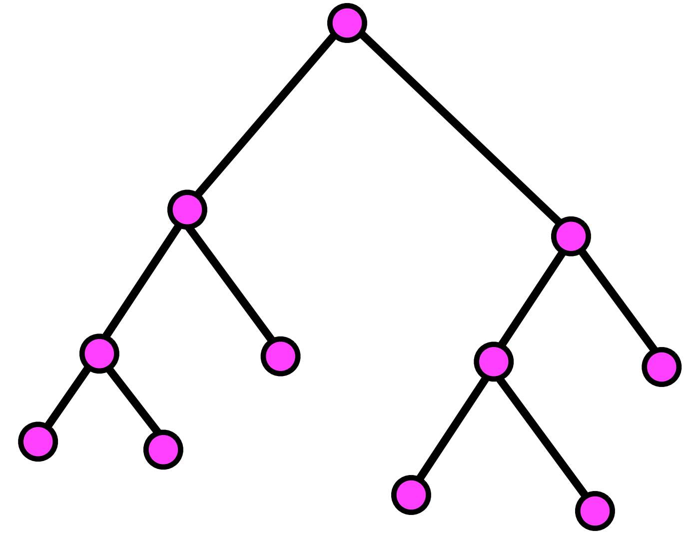


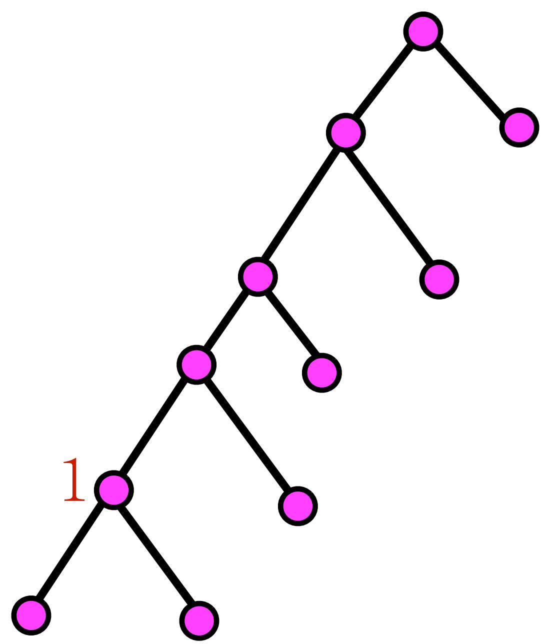


symmetric
numbering of
internal vertices

TREES

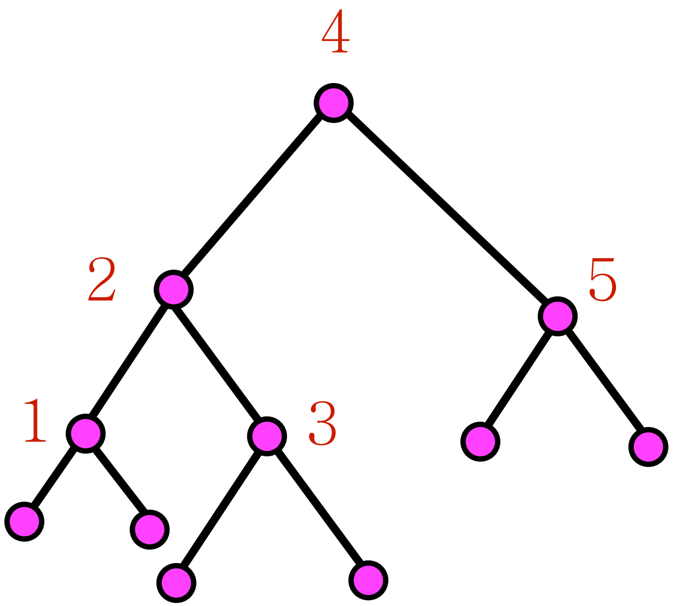


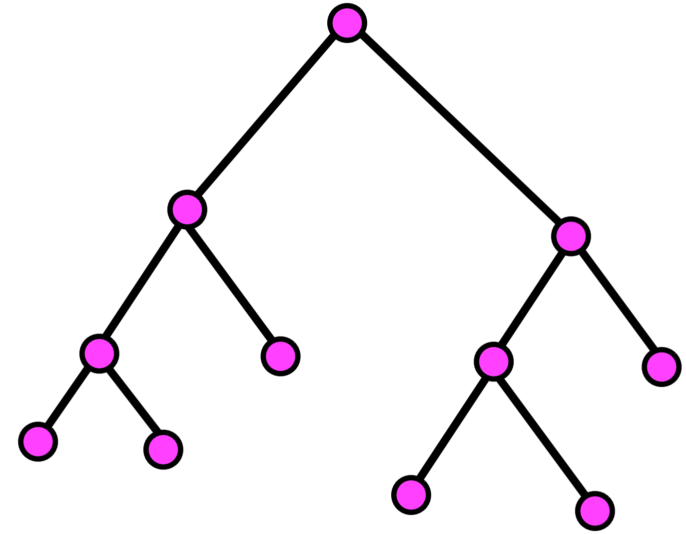


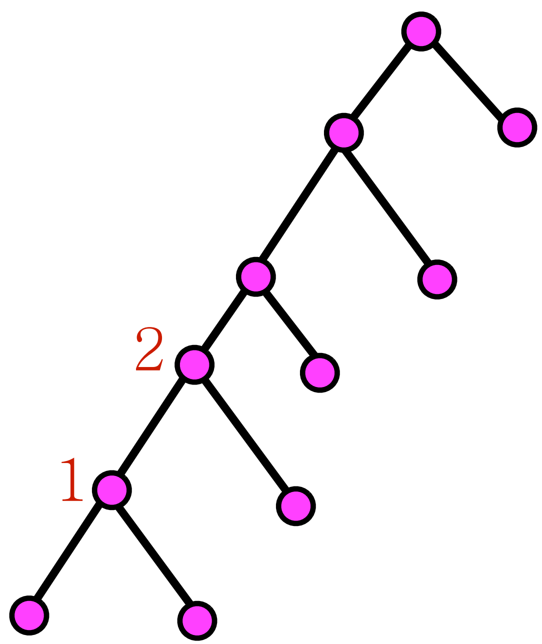


symmetric
numbering of
internal vertices

TREES

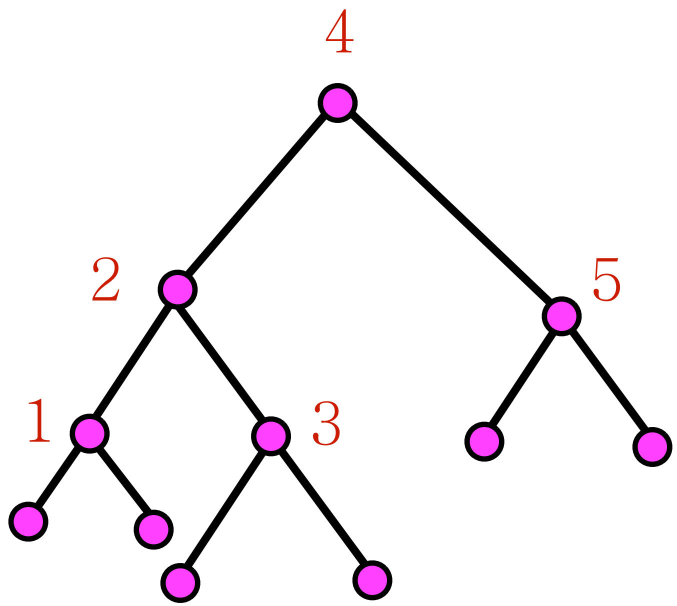


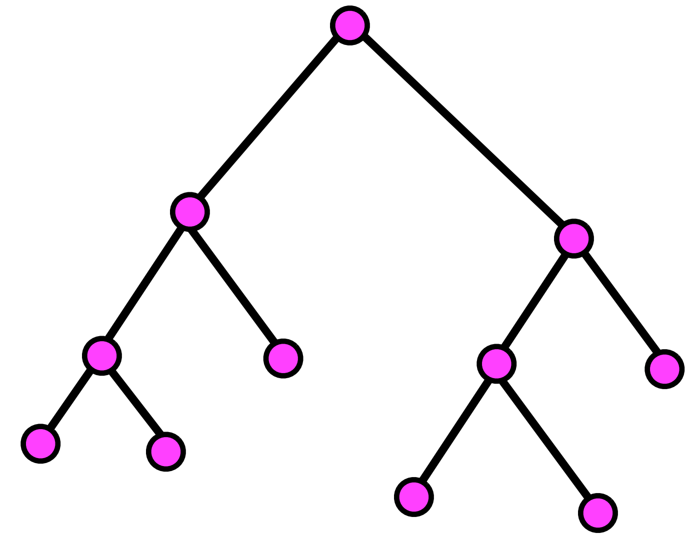




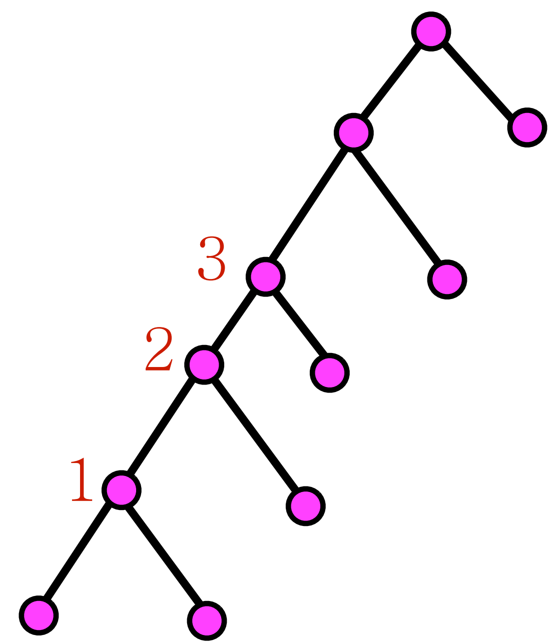
symmetric numbering of internal vertices

TREES





symmetric
numbering of
internal vertices



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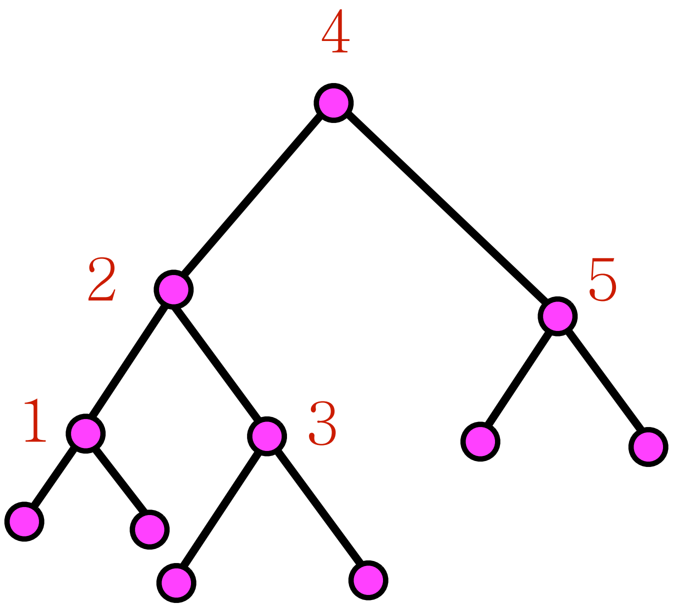
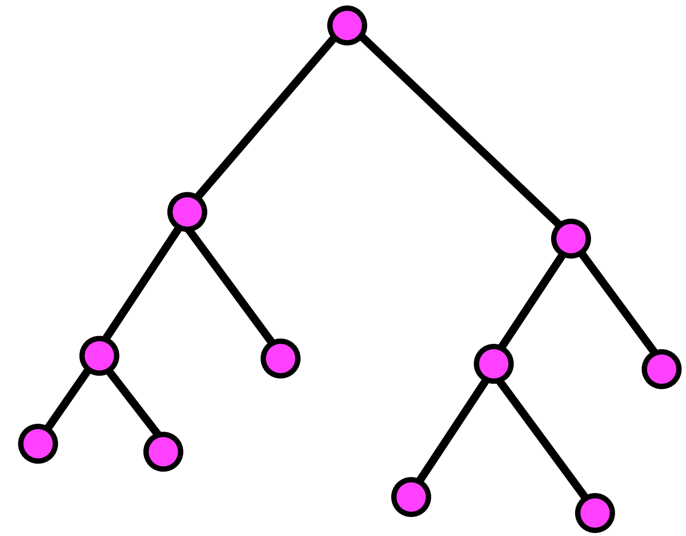
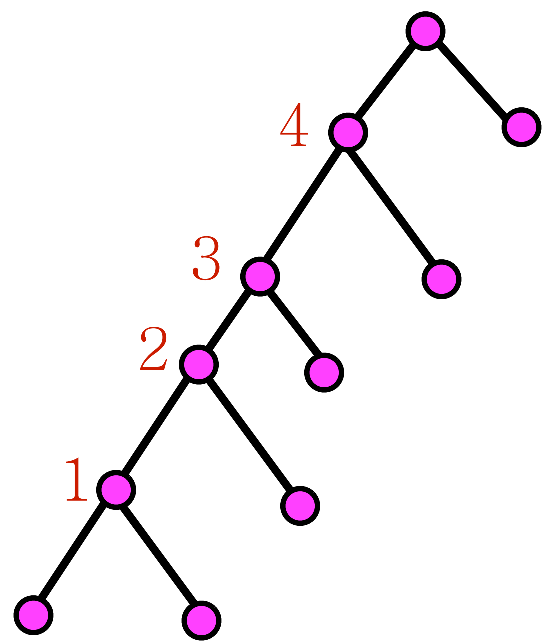
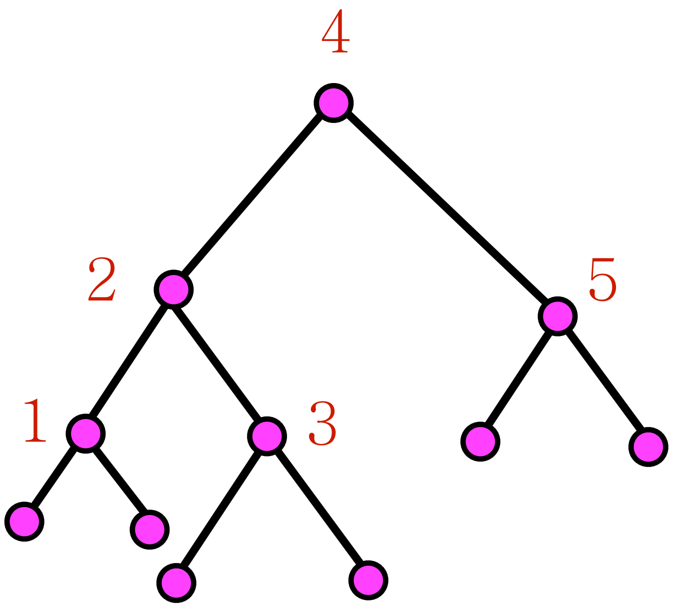


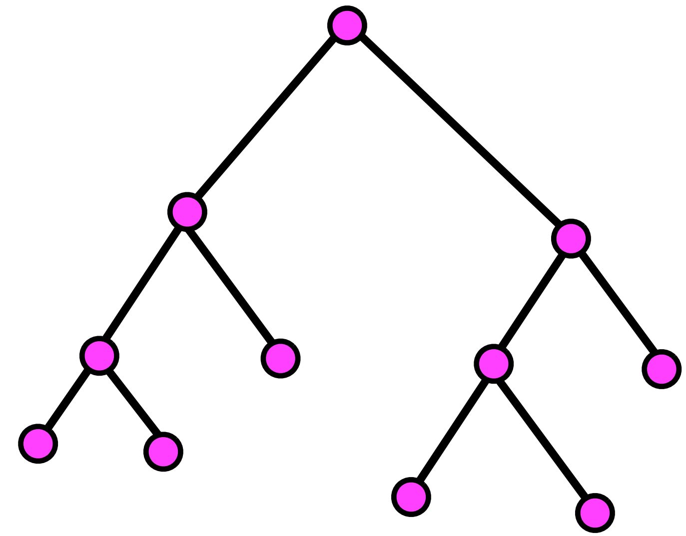




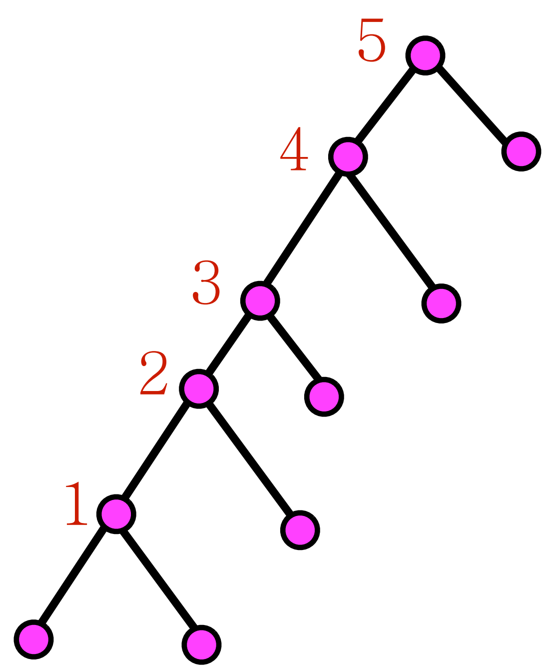
symmetric
numbering of
internal vertices

TREES





symmetric
numbering of
internal vertices



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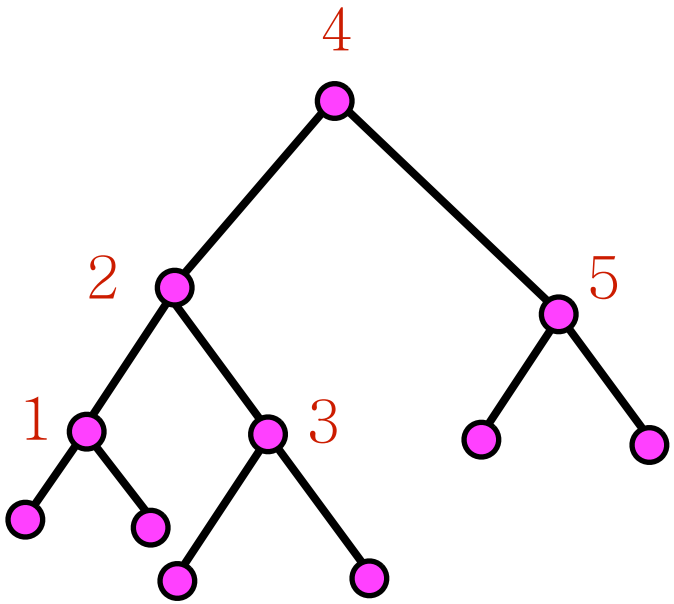
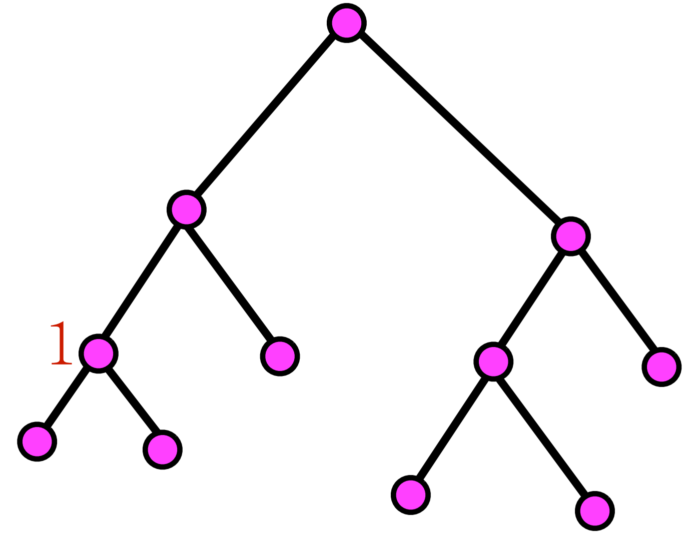
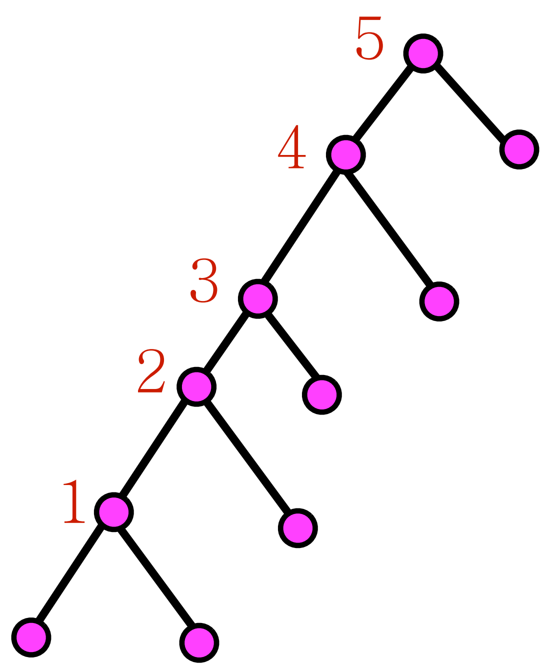
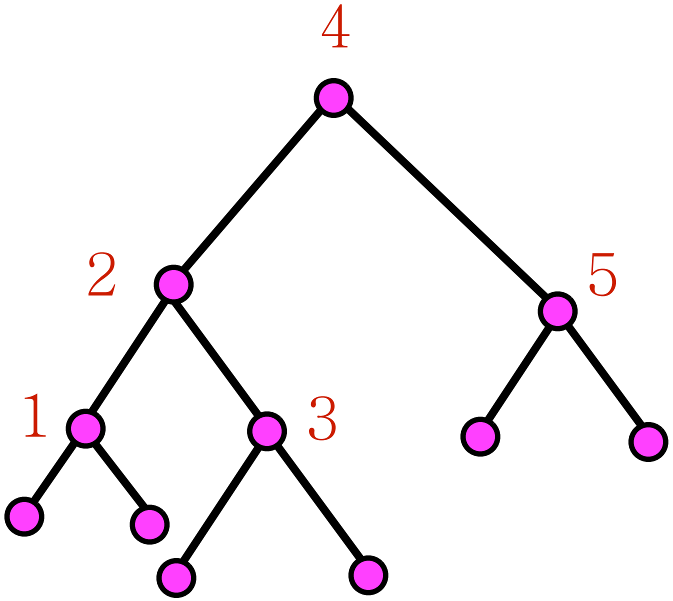


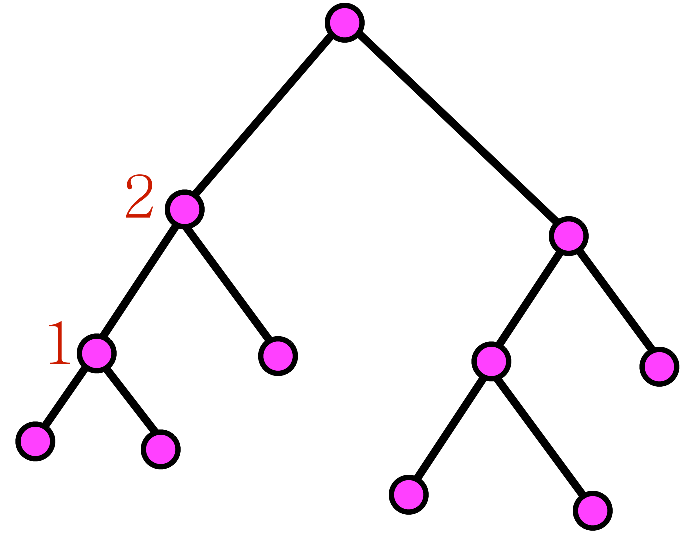


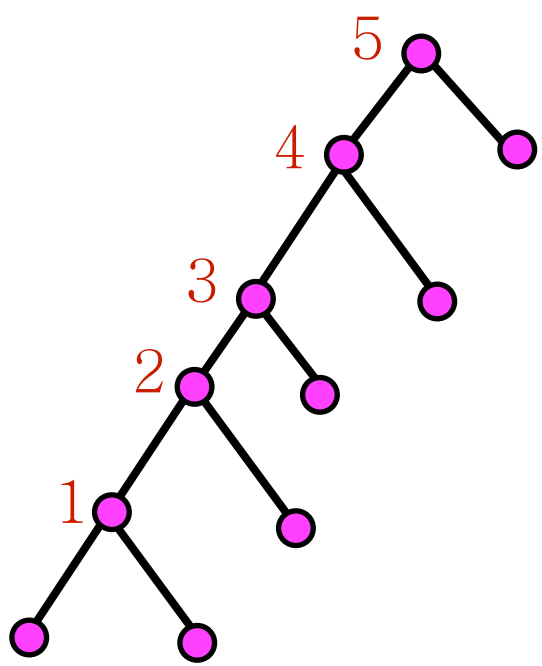


symmetric
numbering of
internal vertices

TREES

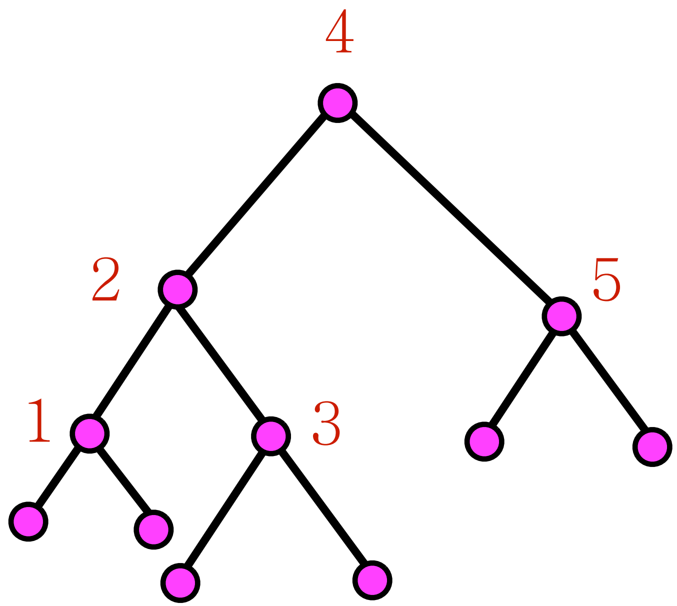


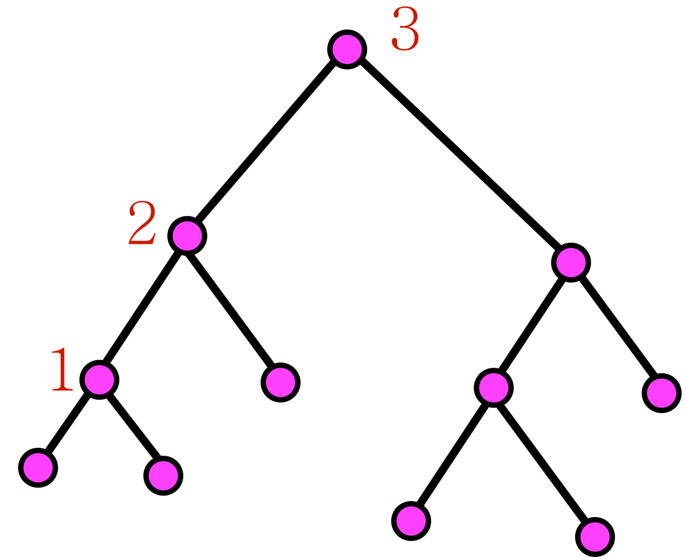


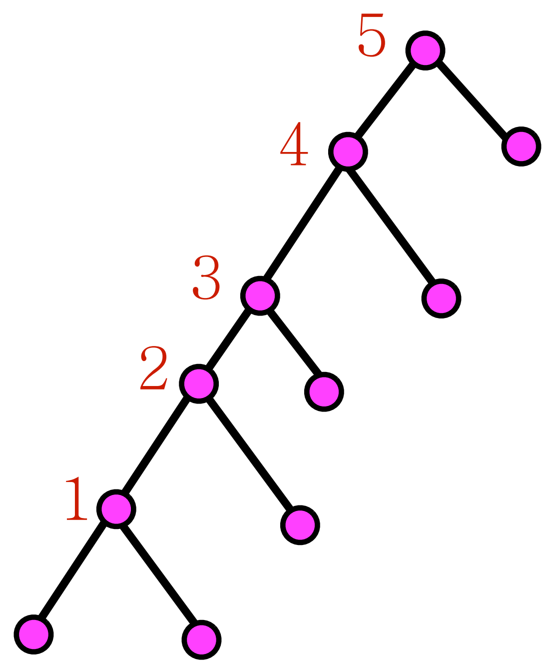


symmetric
numbering of
internal vertices

TREES

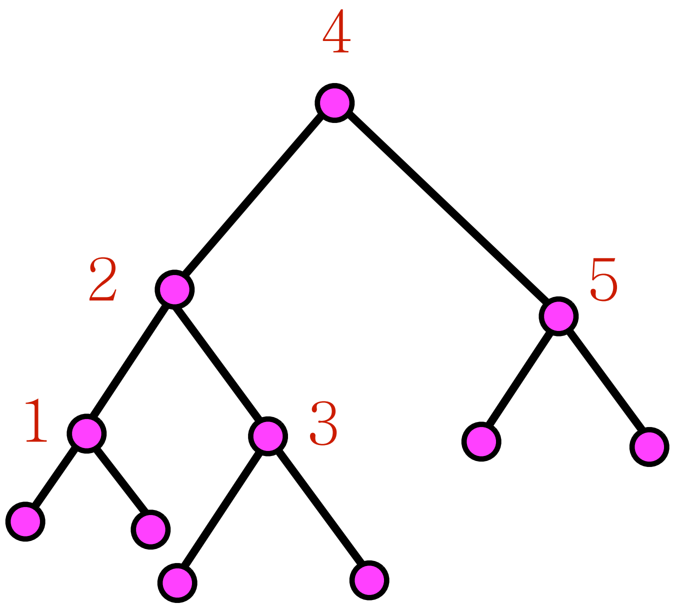


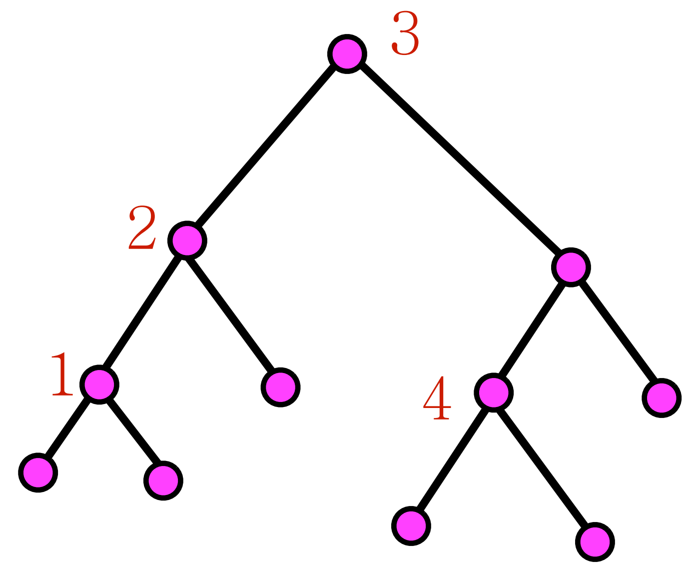


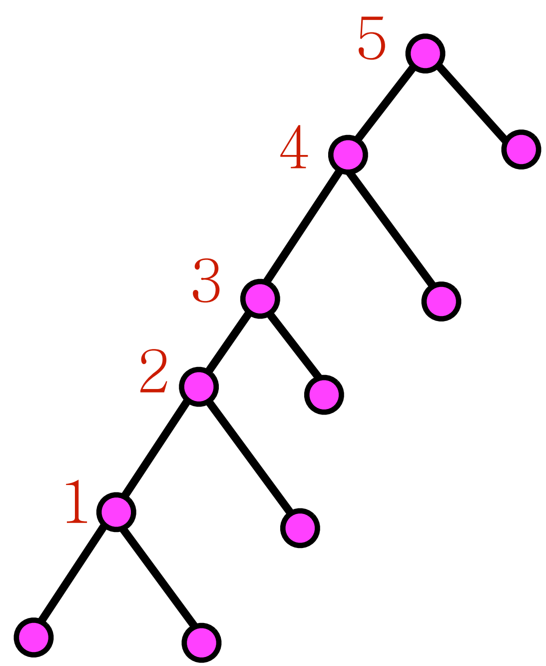


symmetric
numbering of
internal vertices

TREES

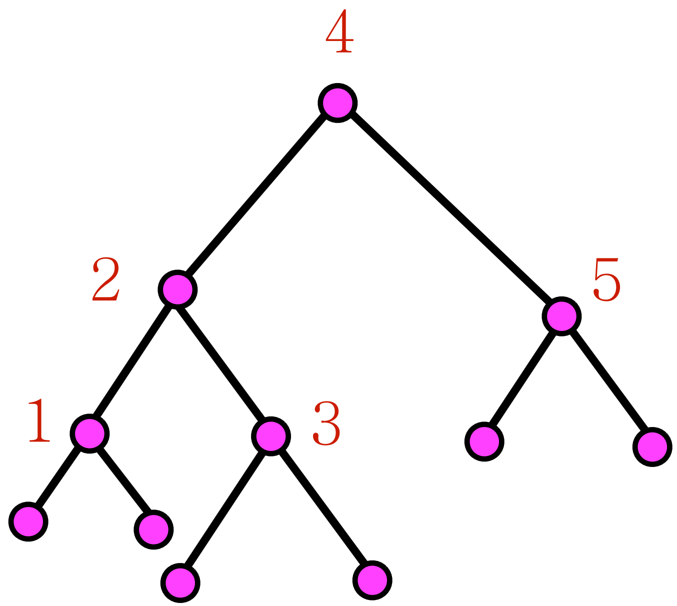


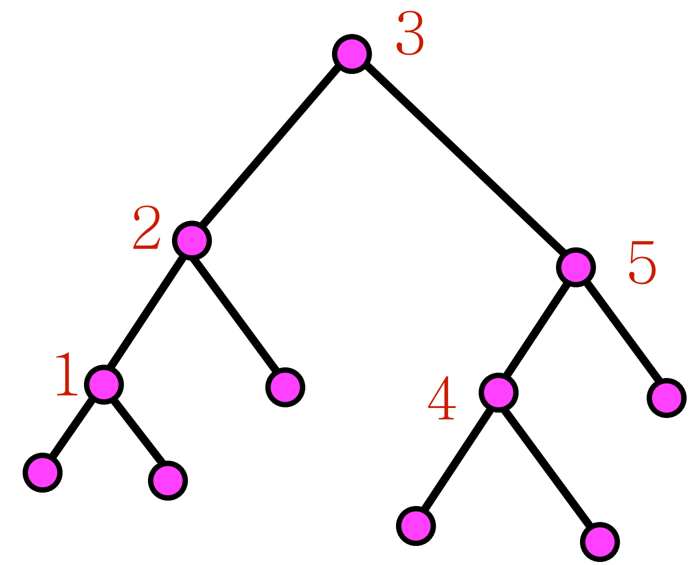


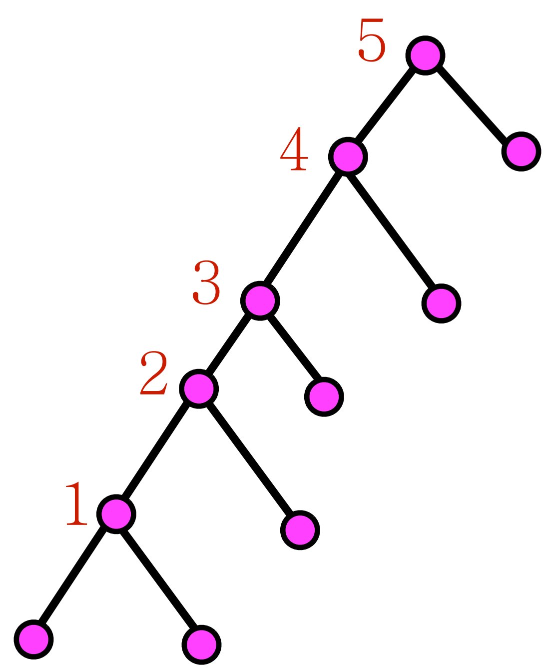


symmetric
numbering of
internal vertices

TREES

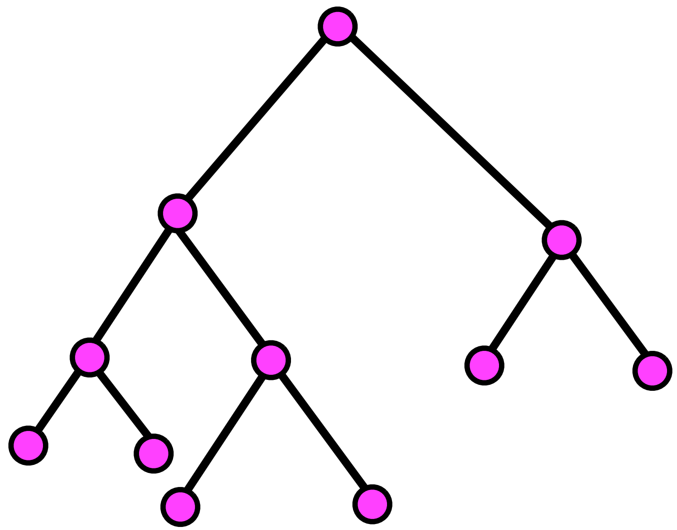


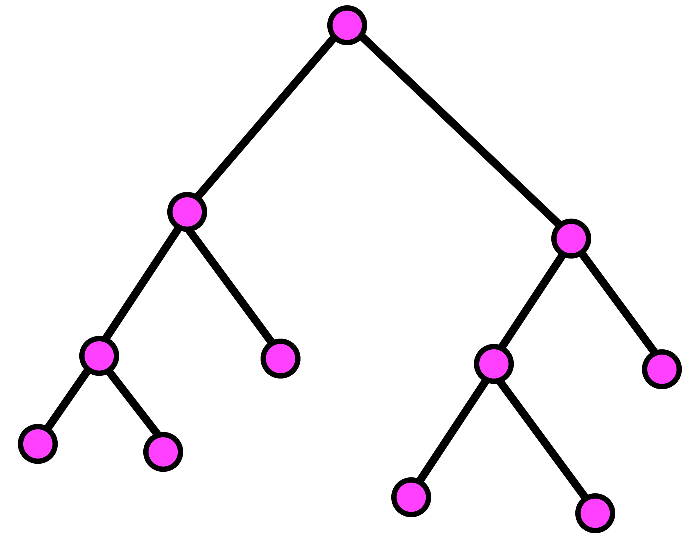


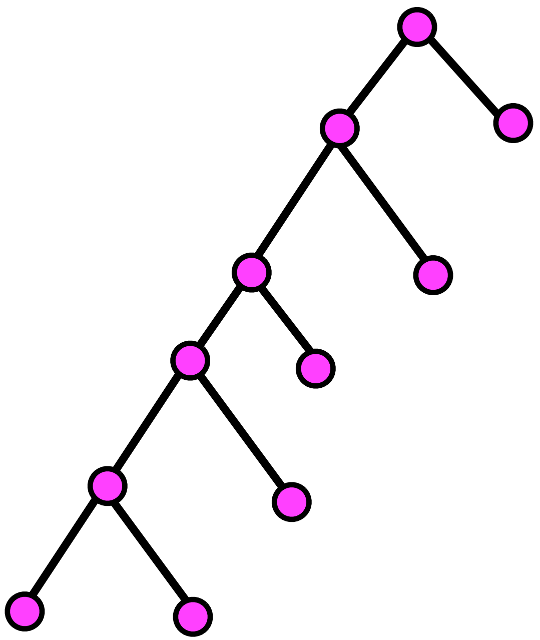


symmetric numbering of internal vertices

TREES







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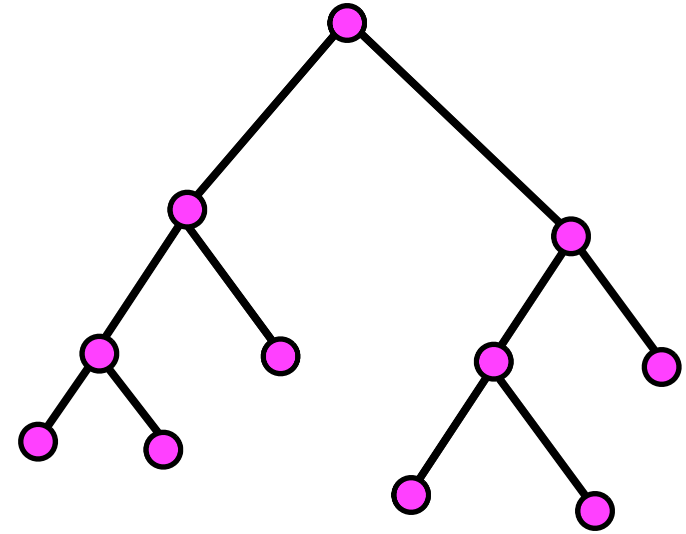

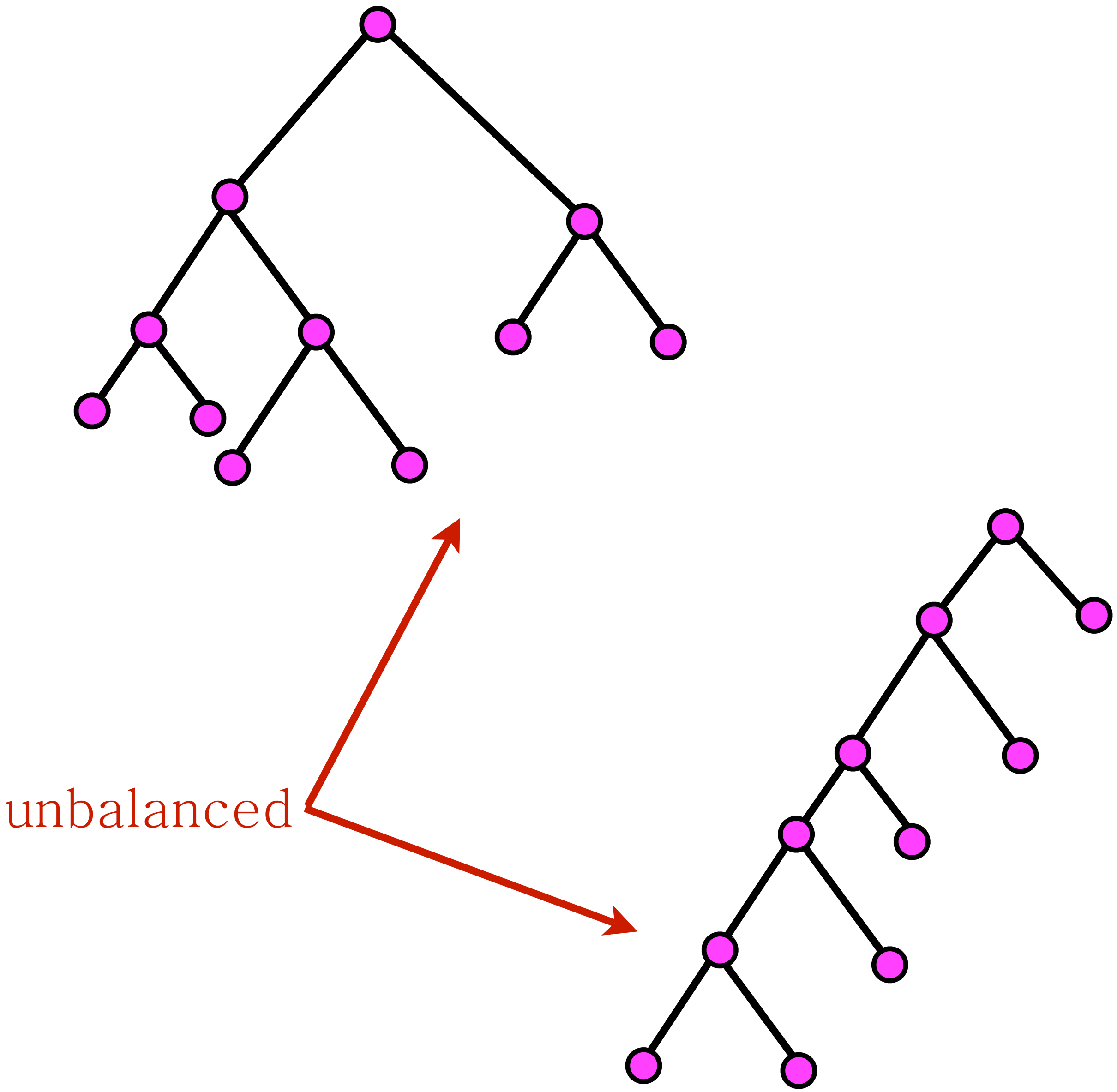

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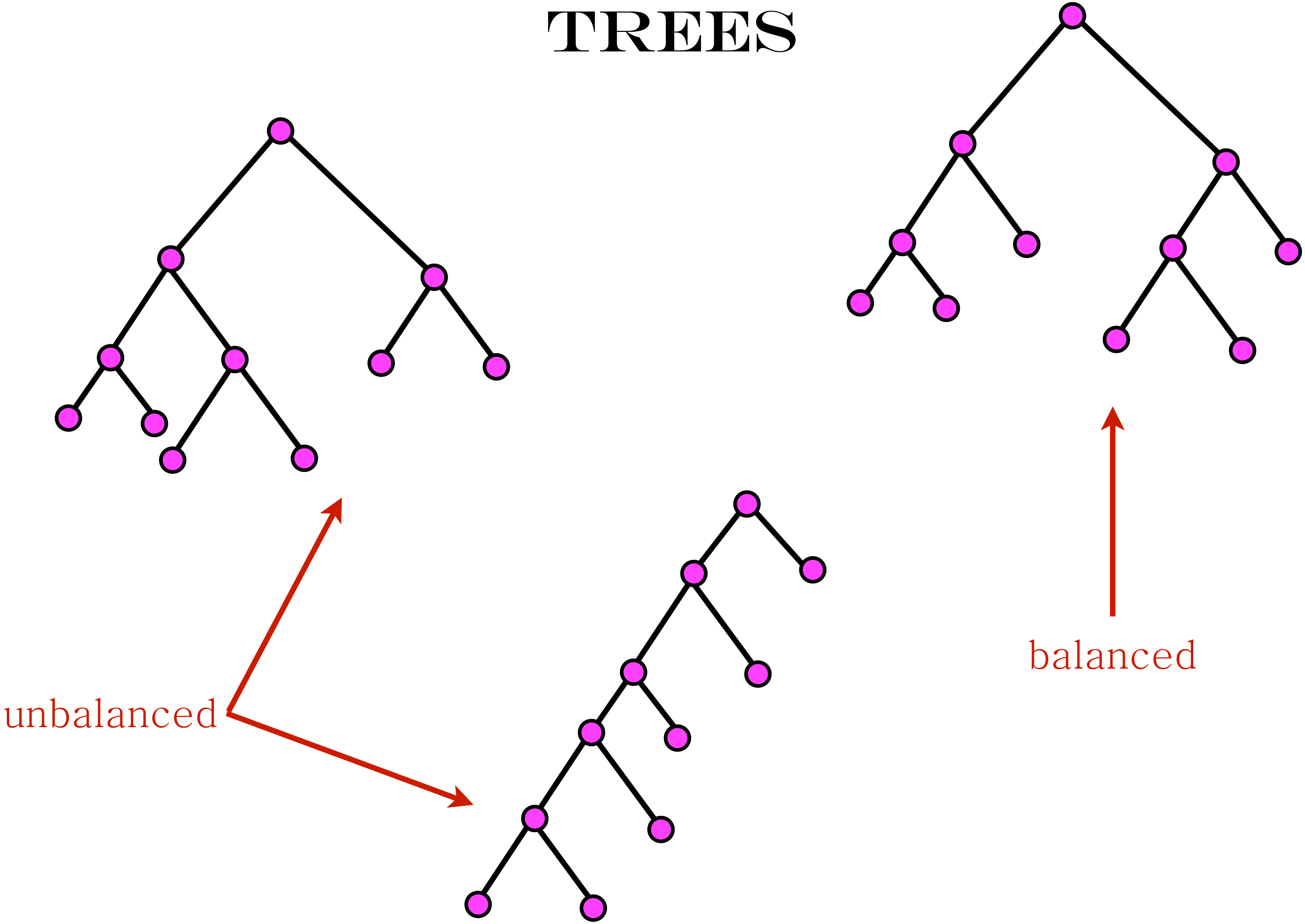

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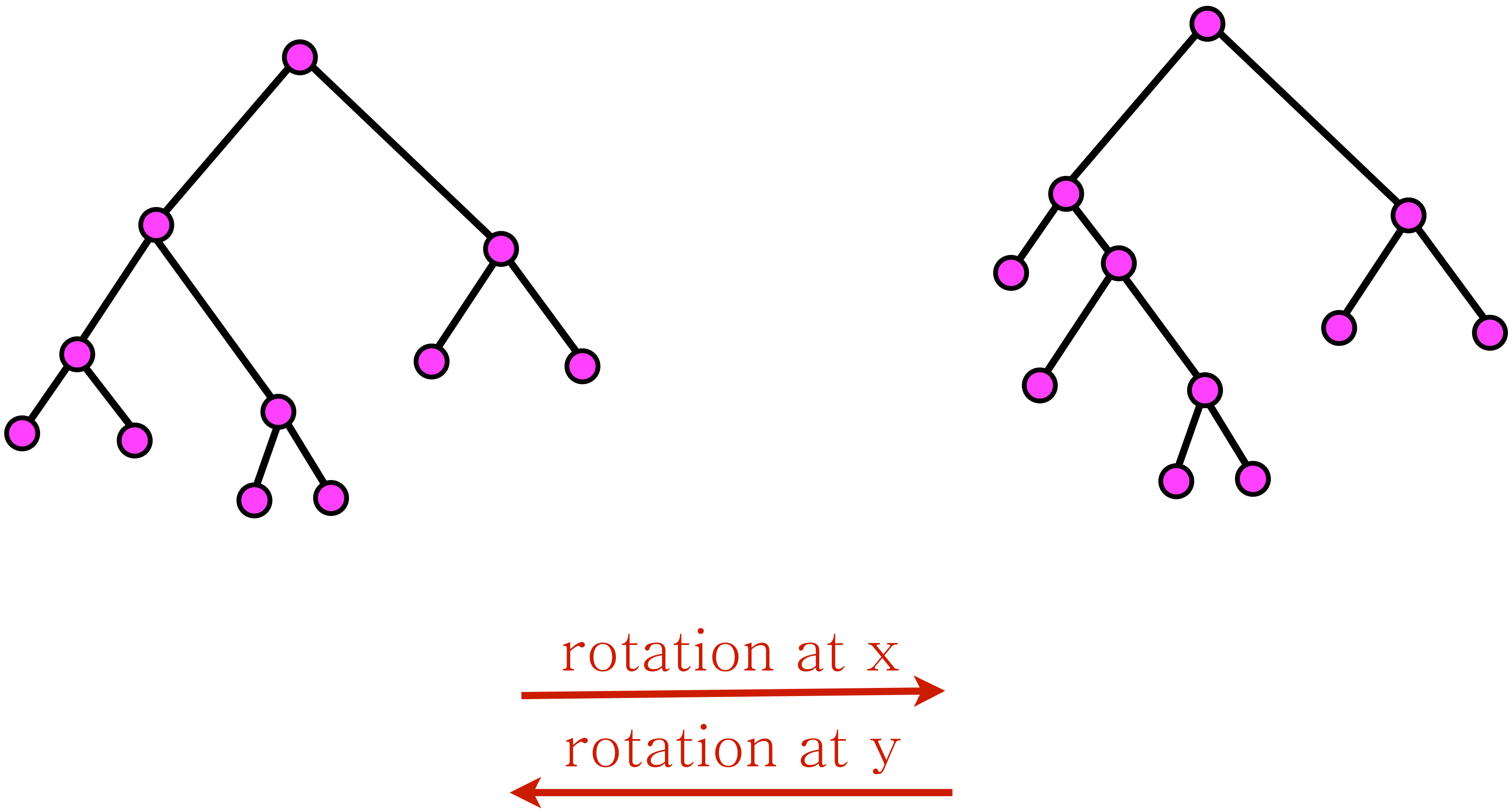

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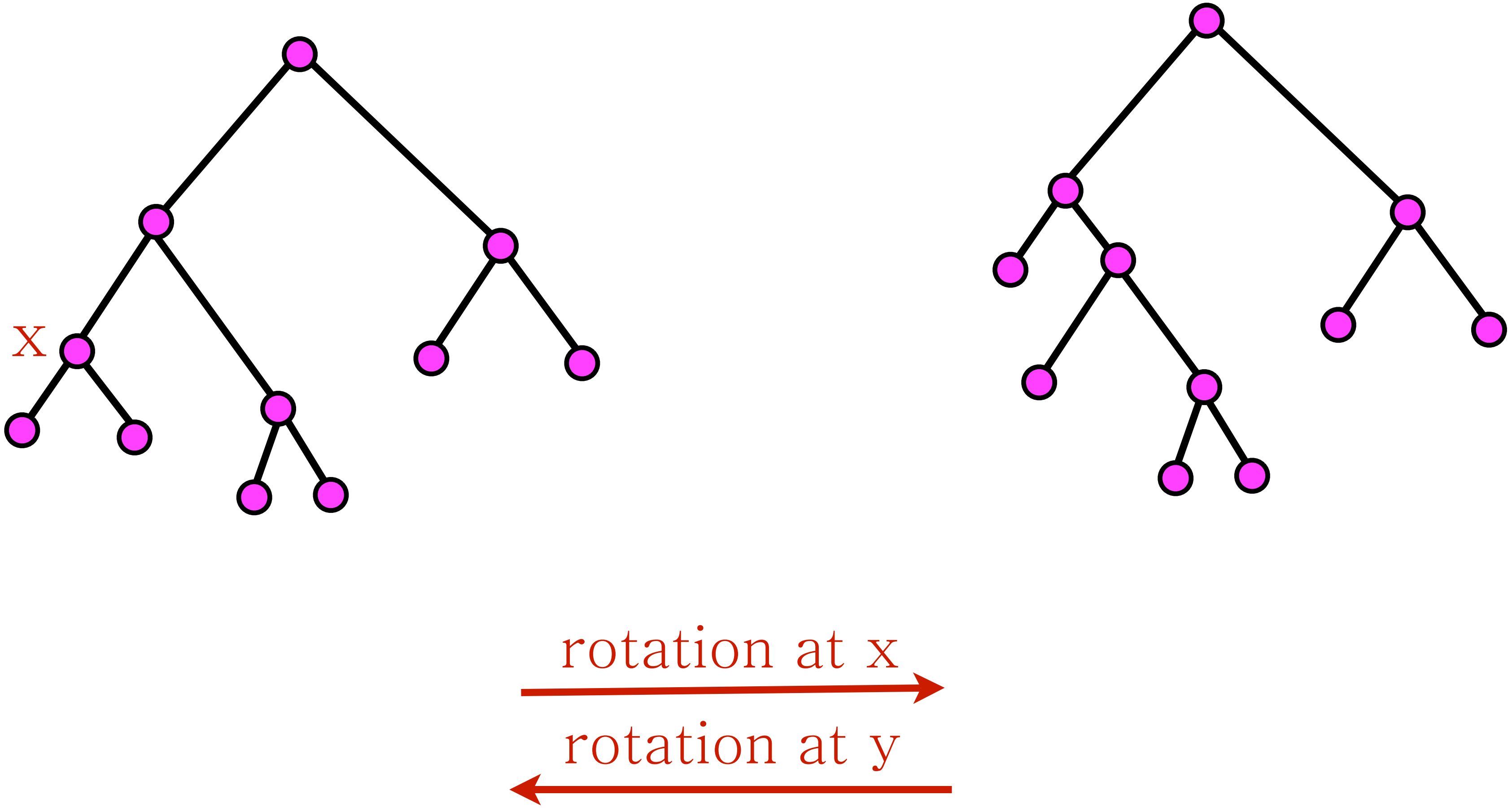

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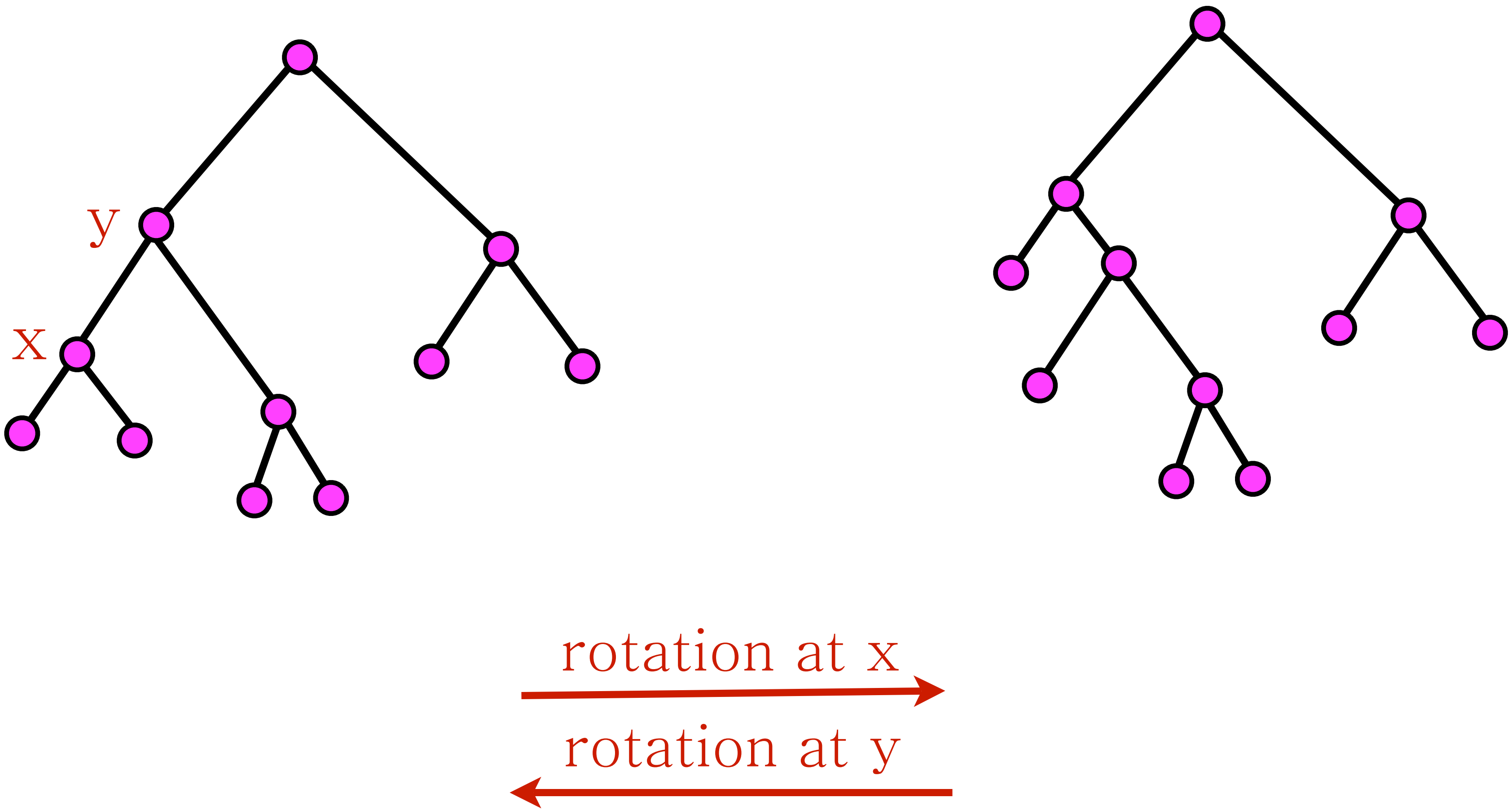

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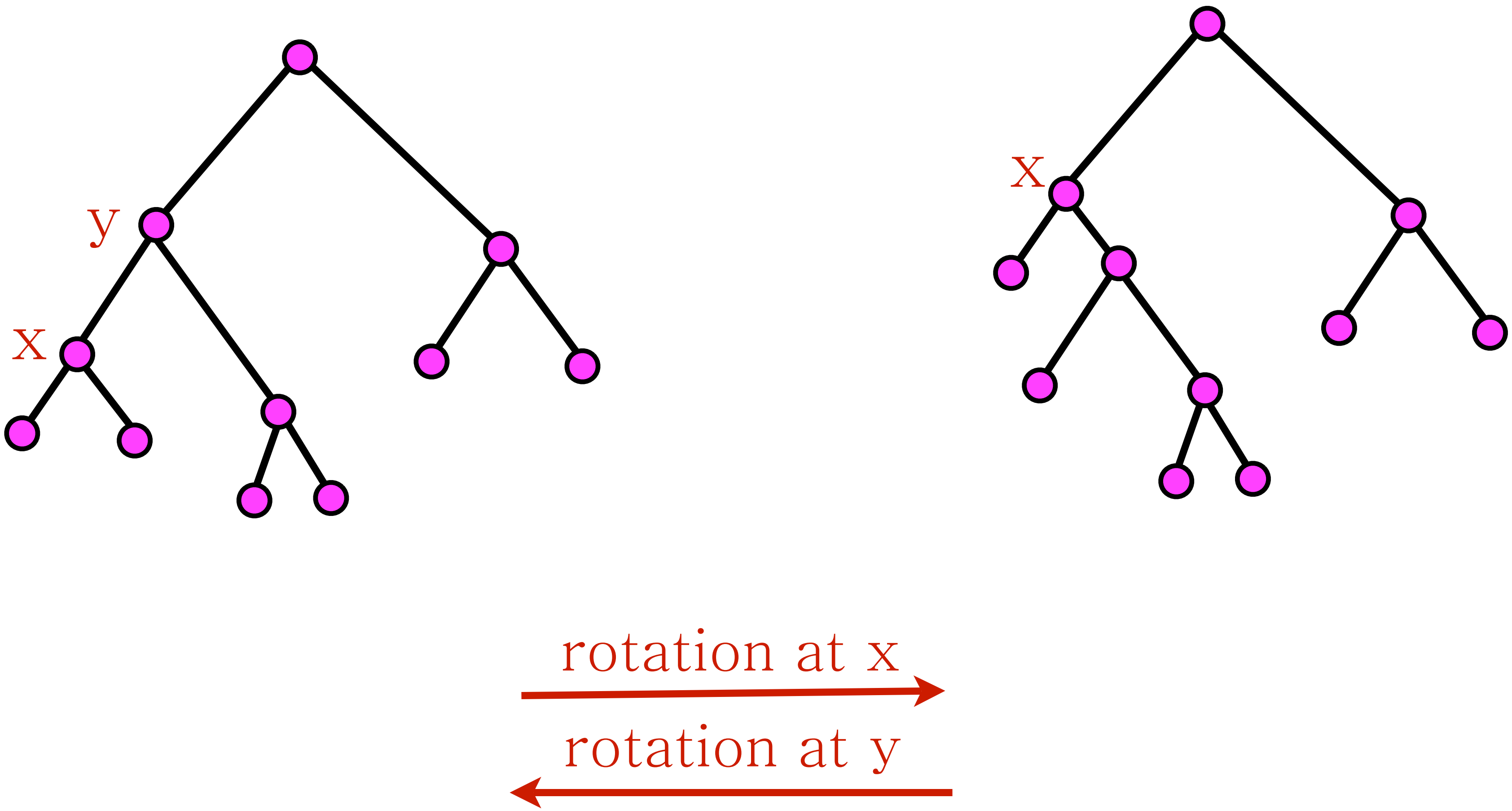

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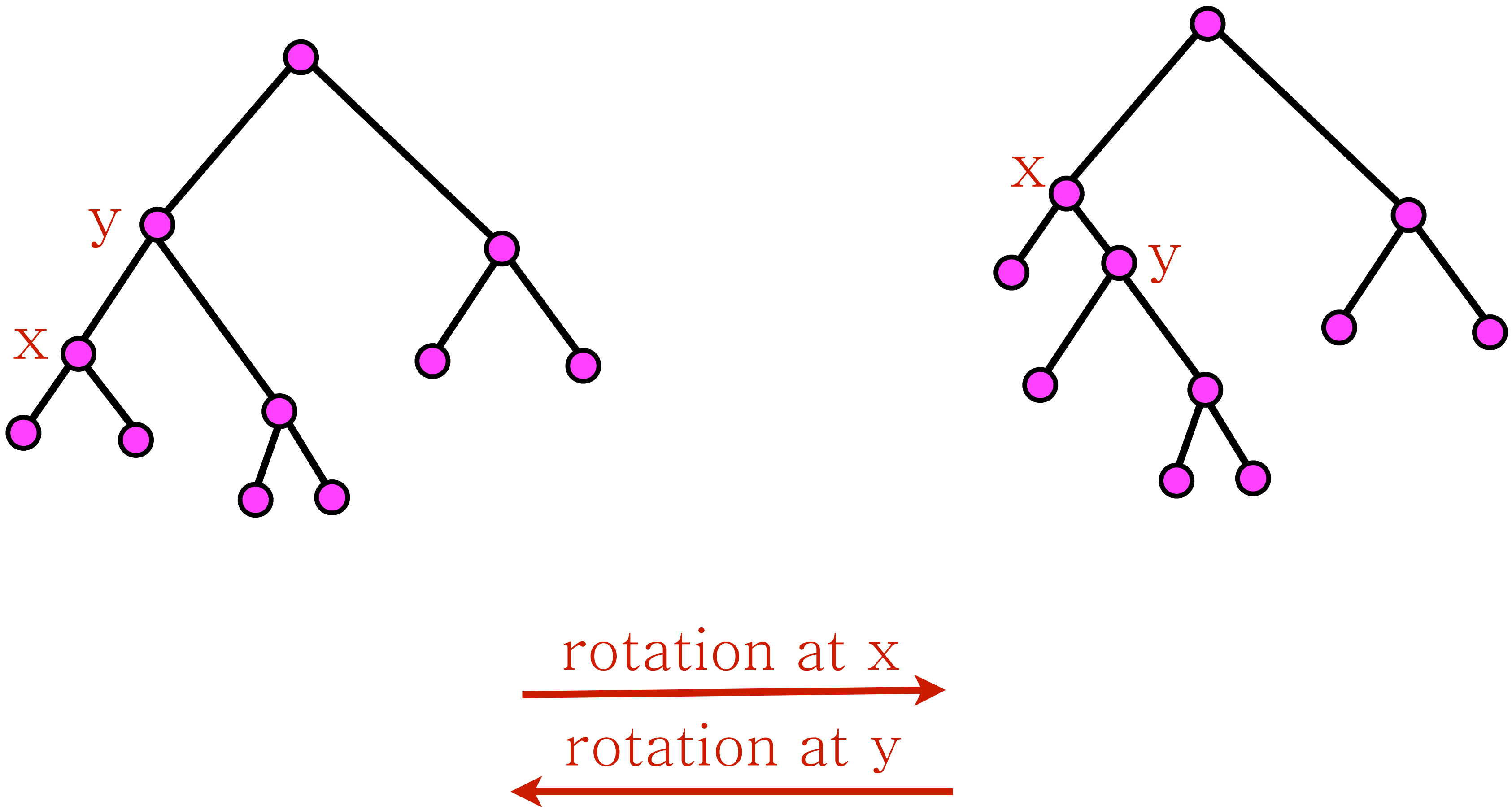

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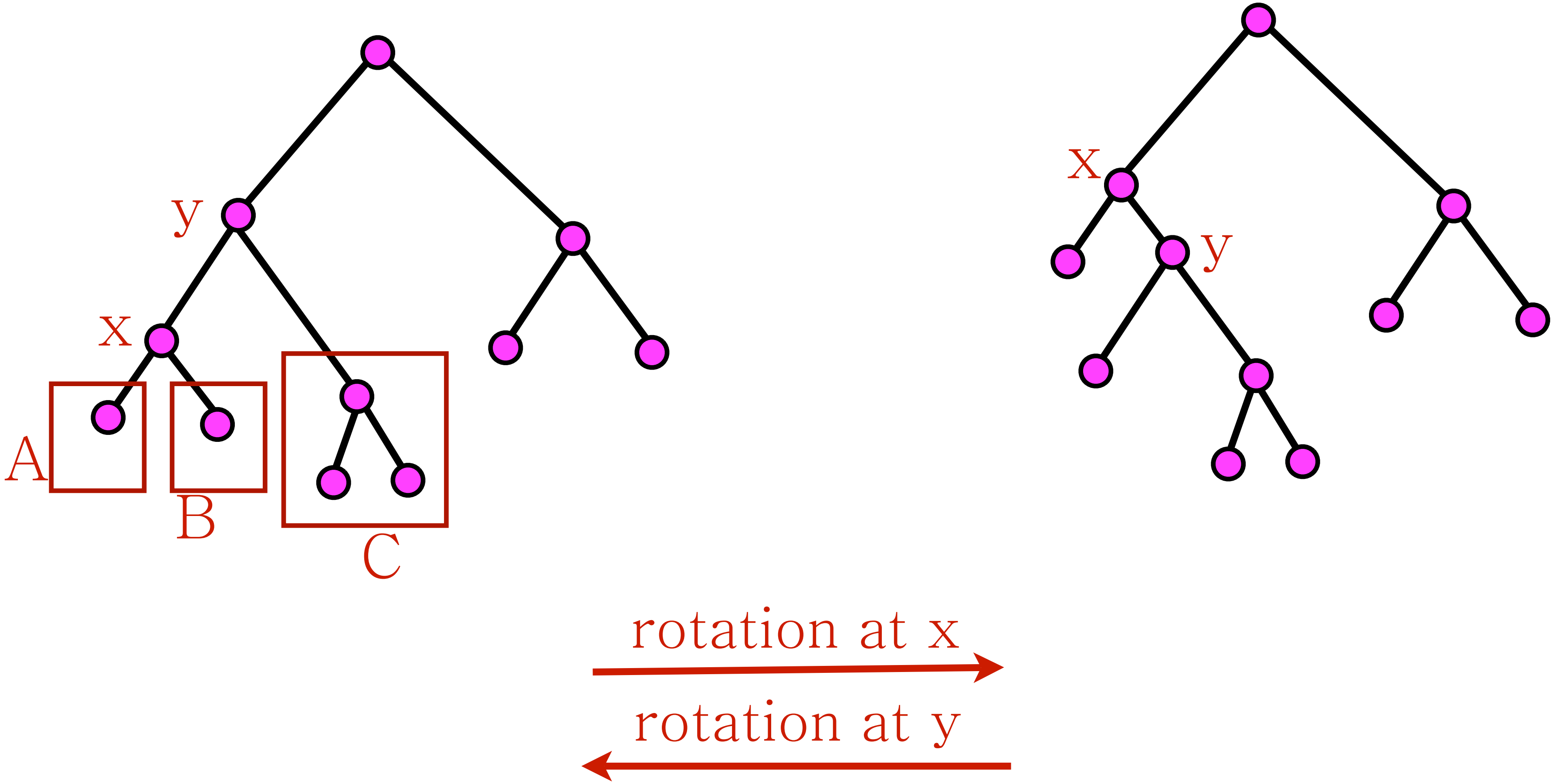

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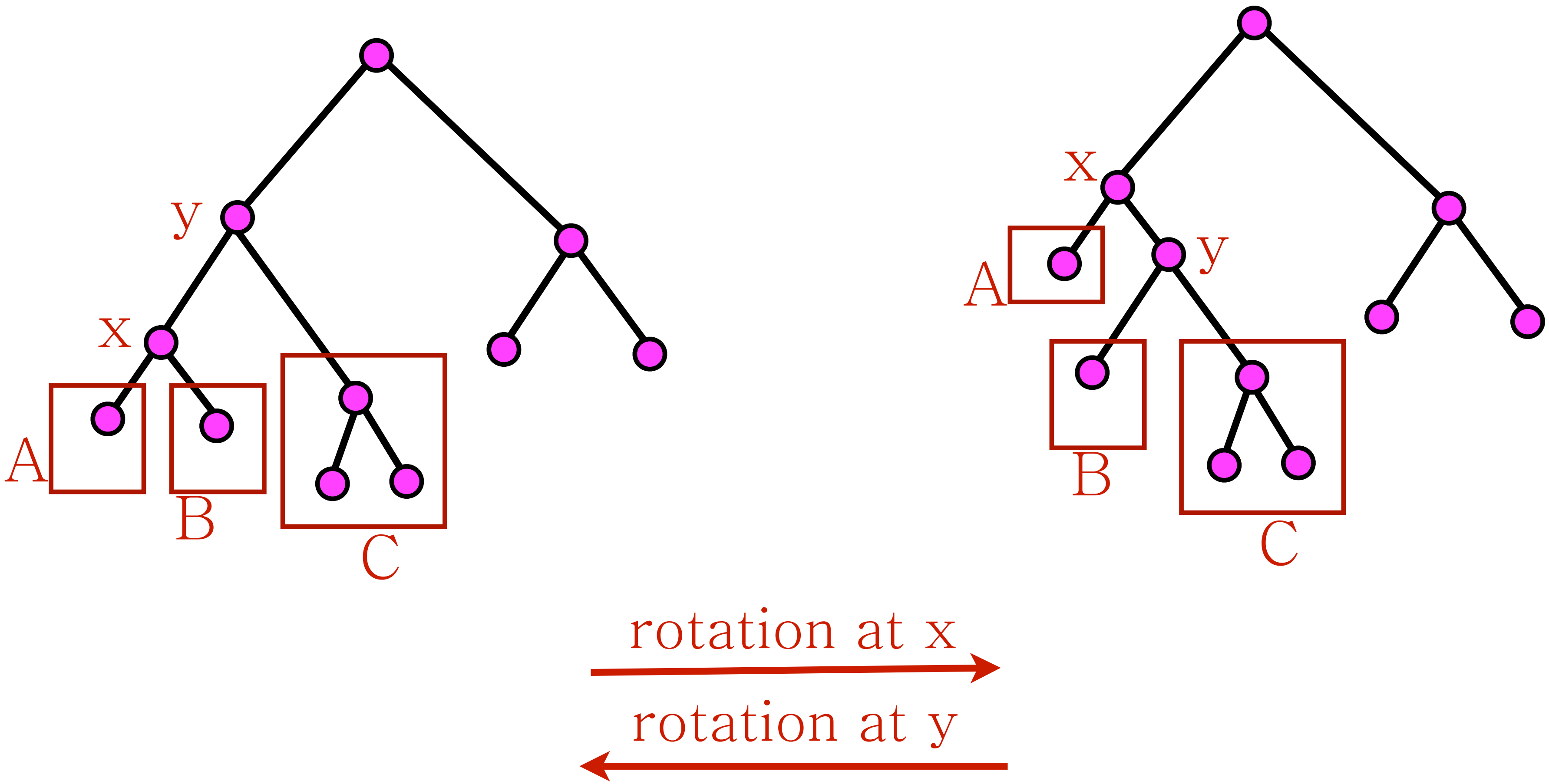
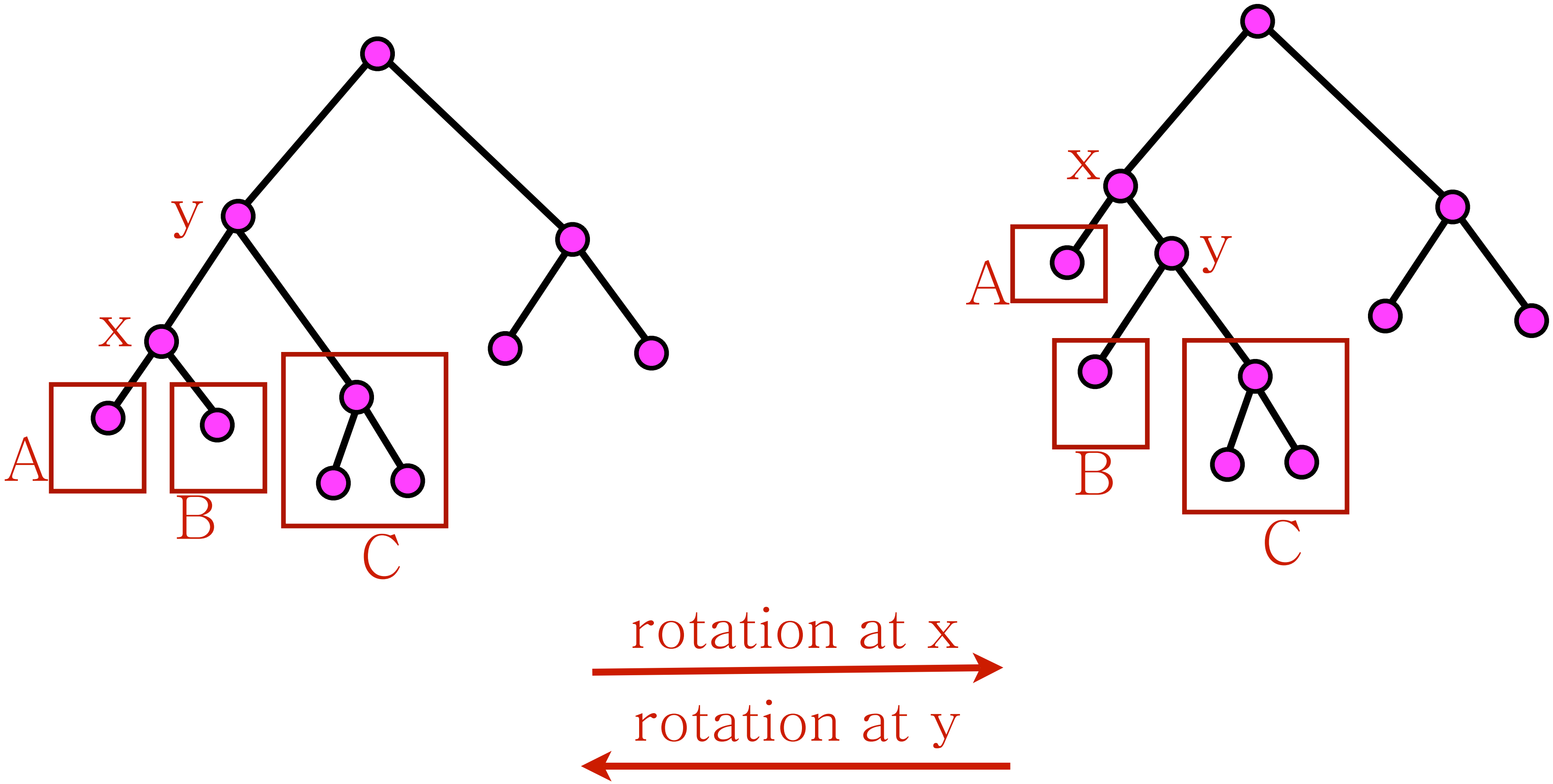
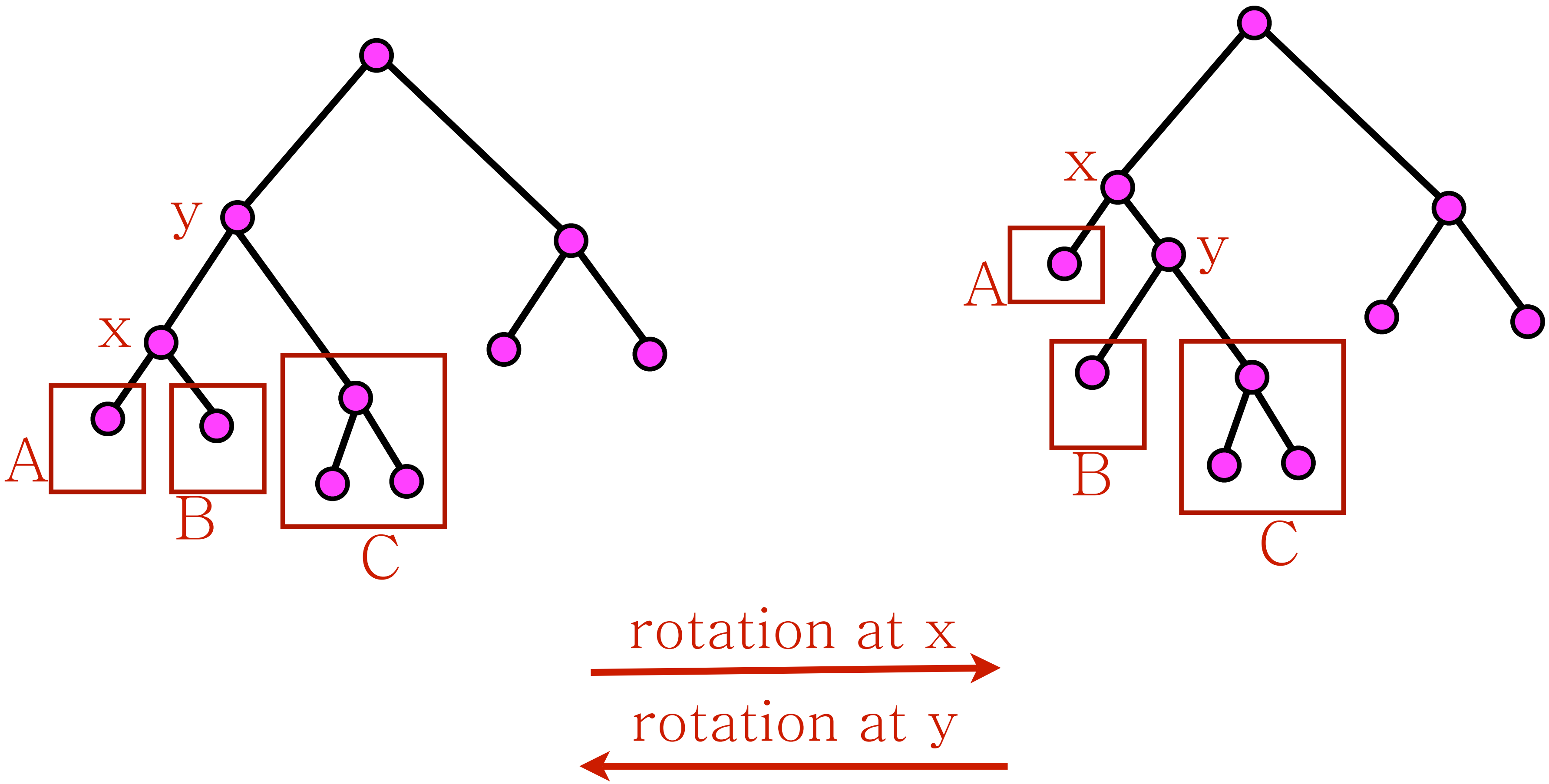


TREES



QUESTION: What is the maximal number of rotations needed to convert between two binary trees of size n?

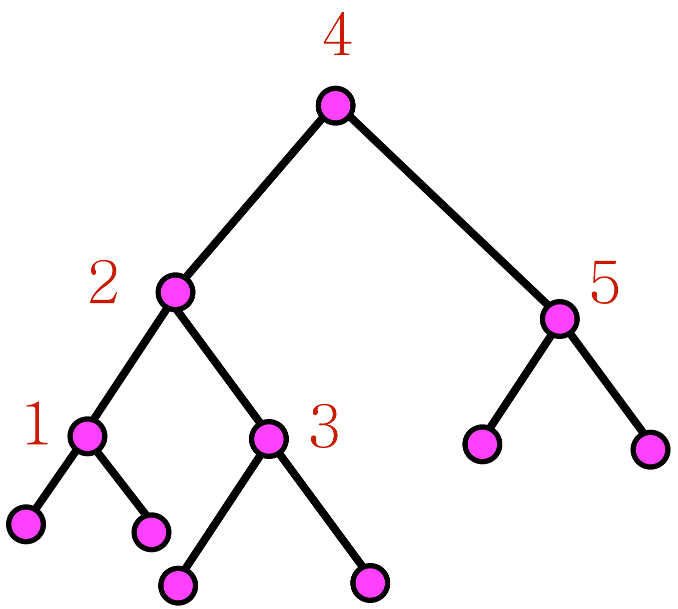
TREES



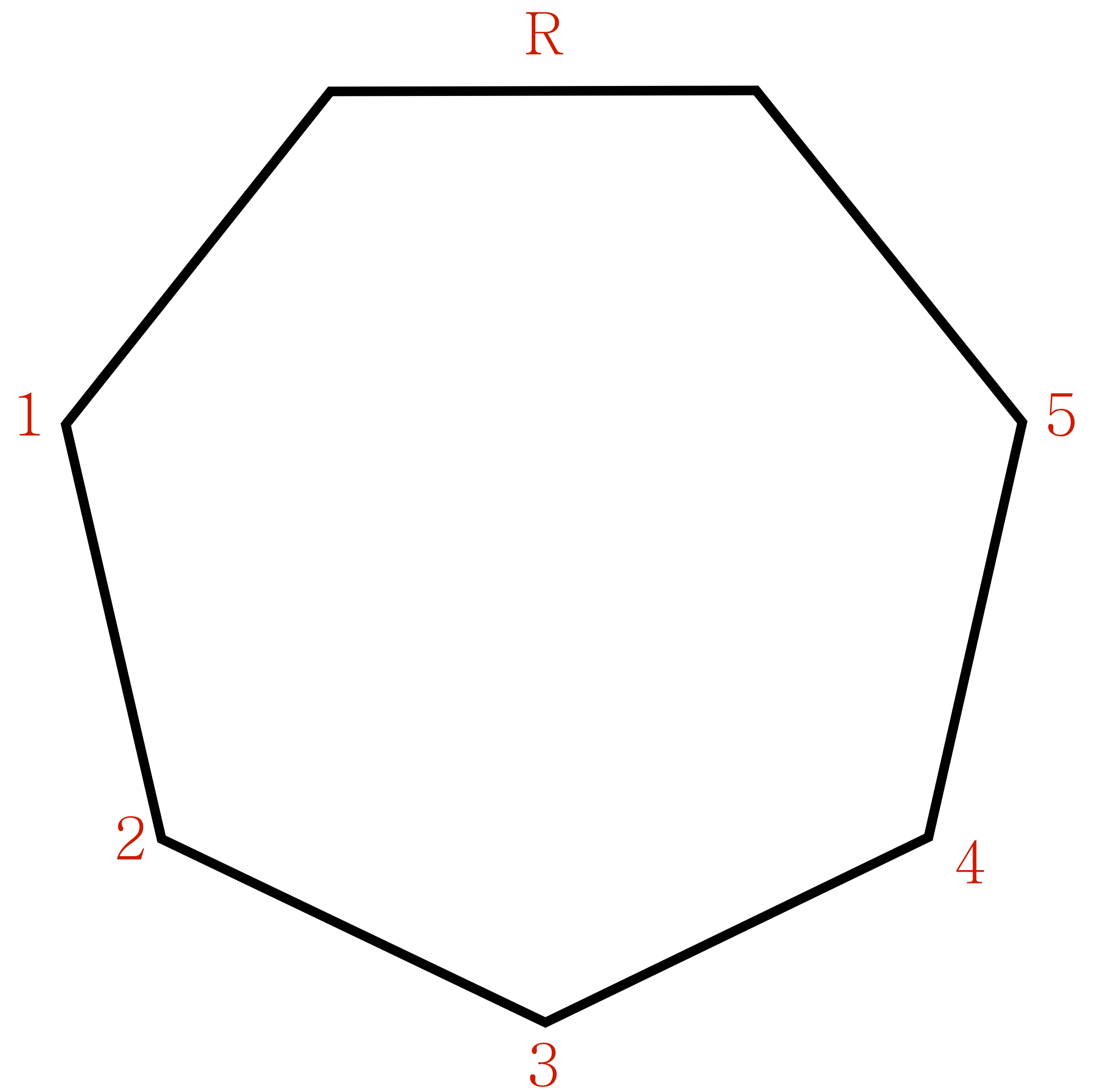
QUESTION: What is the maximal number of rotations needed to convert between two binary trees of size n?

d(n)

POLYGONS

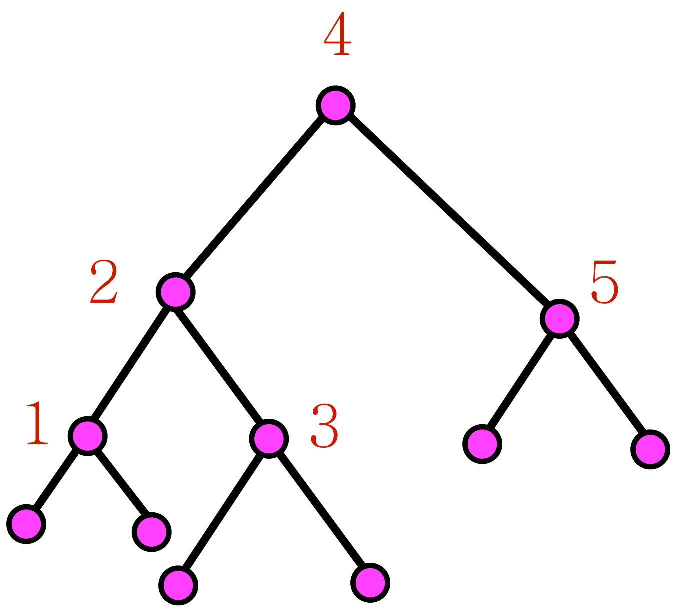


Size n
binary tree

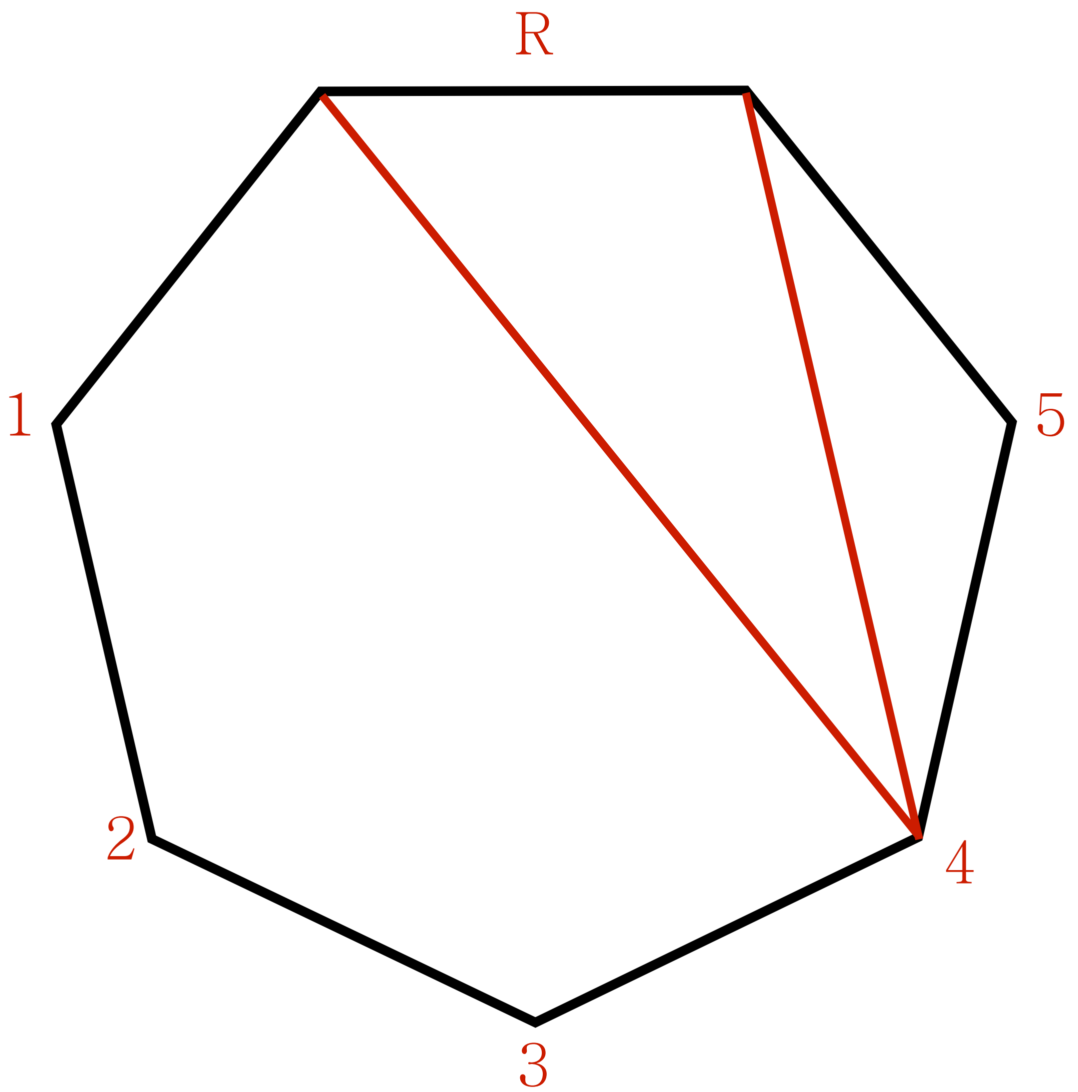


(n+ 2)-gon

POLYGONS

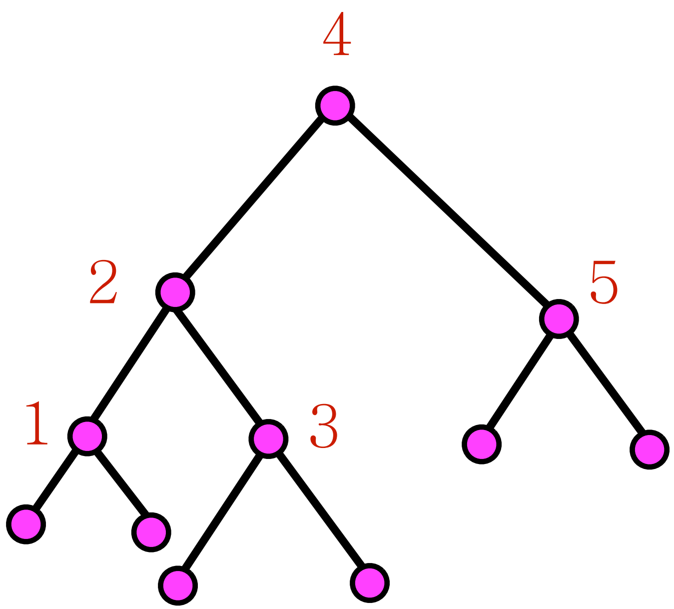


Size n
binary tree

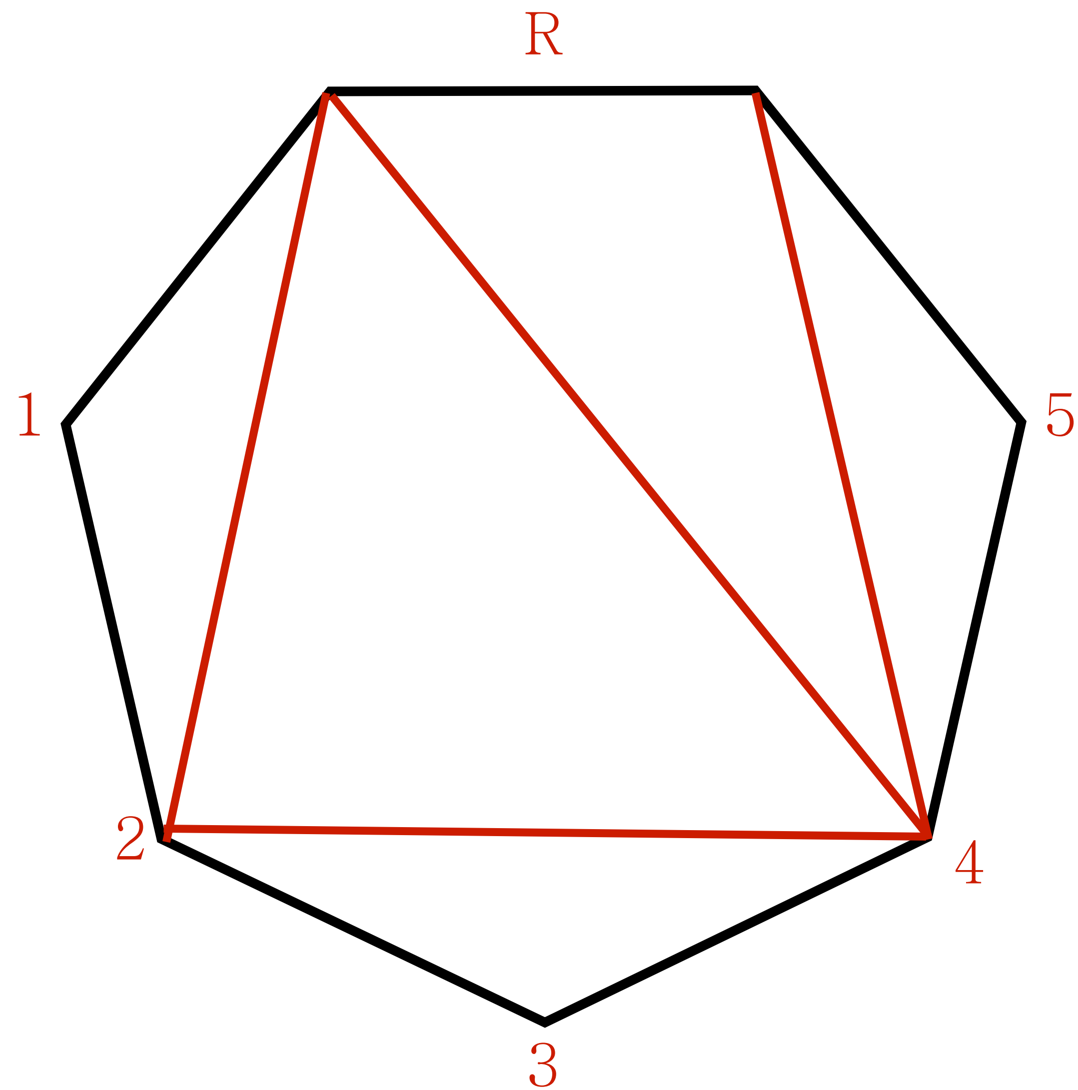


(n+2)-gon

POLYGONS

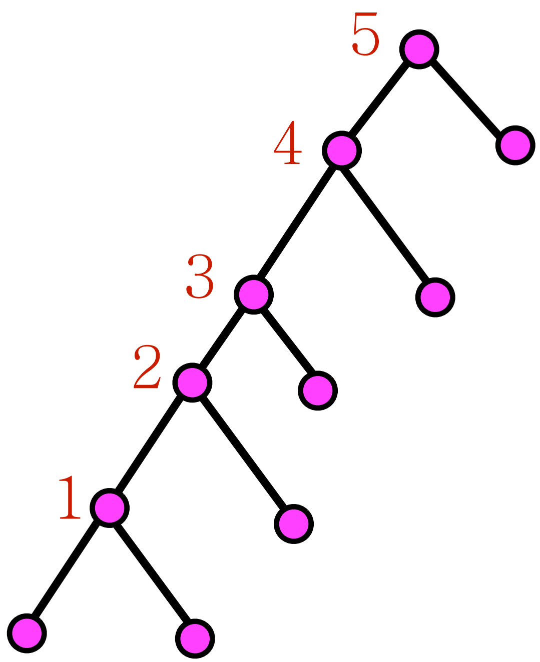


Size n
binary tree

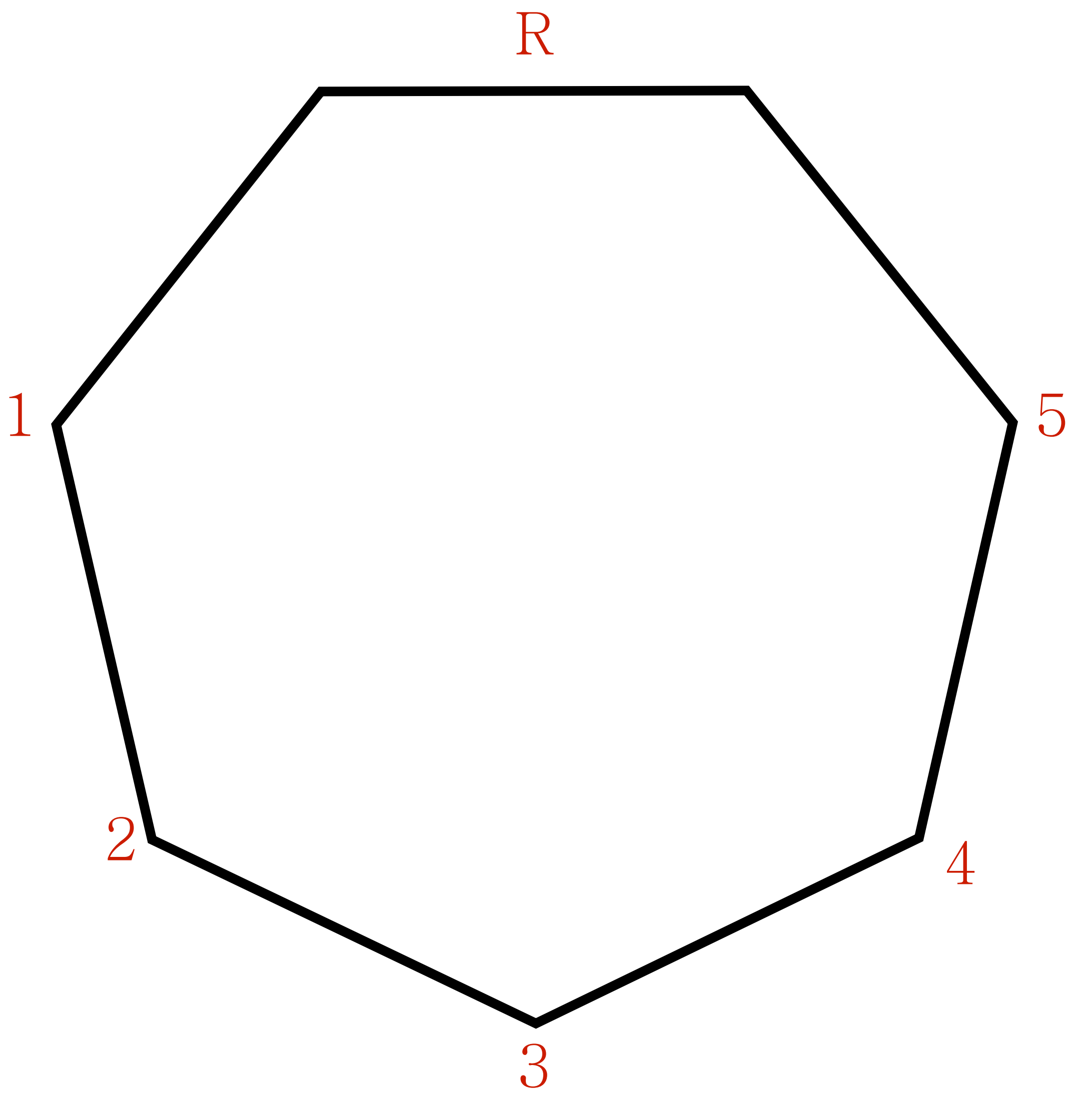


(n+ 2)-gon

POLYGONS

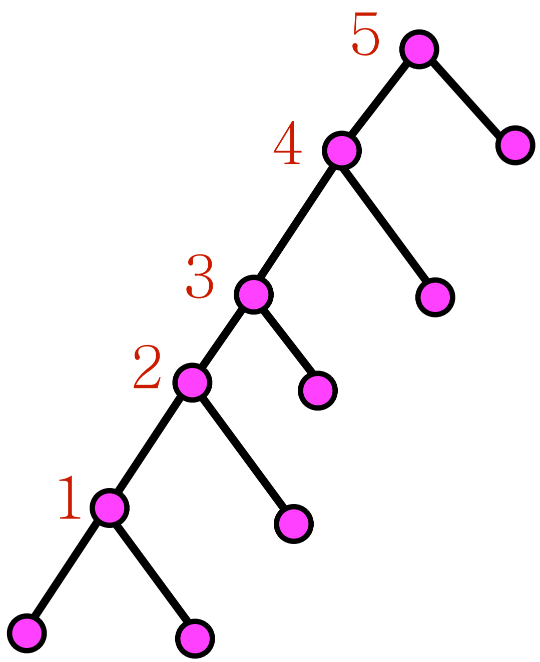


Size n
binary tree

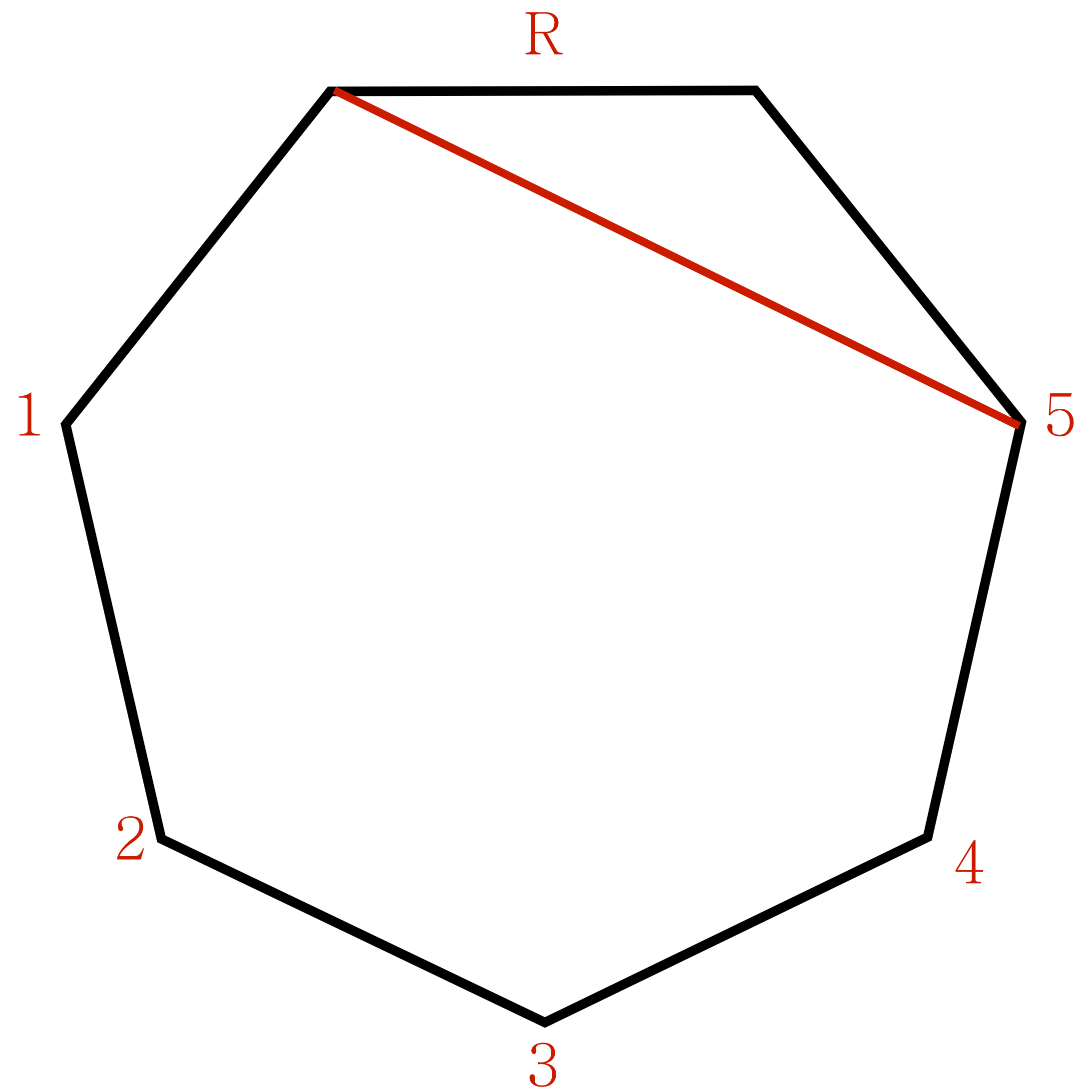


(n+ 2)-gon

POLYGONS

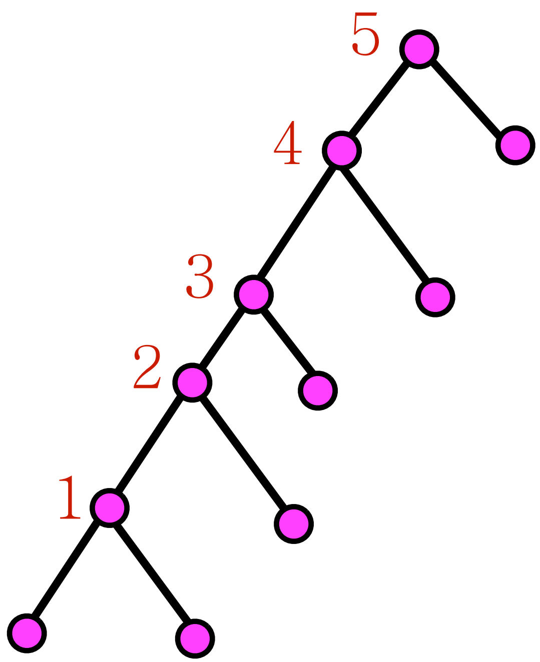


Size n
binary tree



(n+ 2)-gon

POLYGONS



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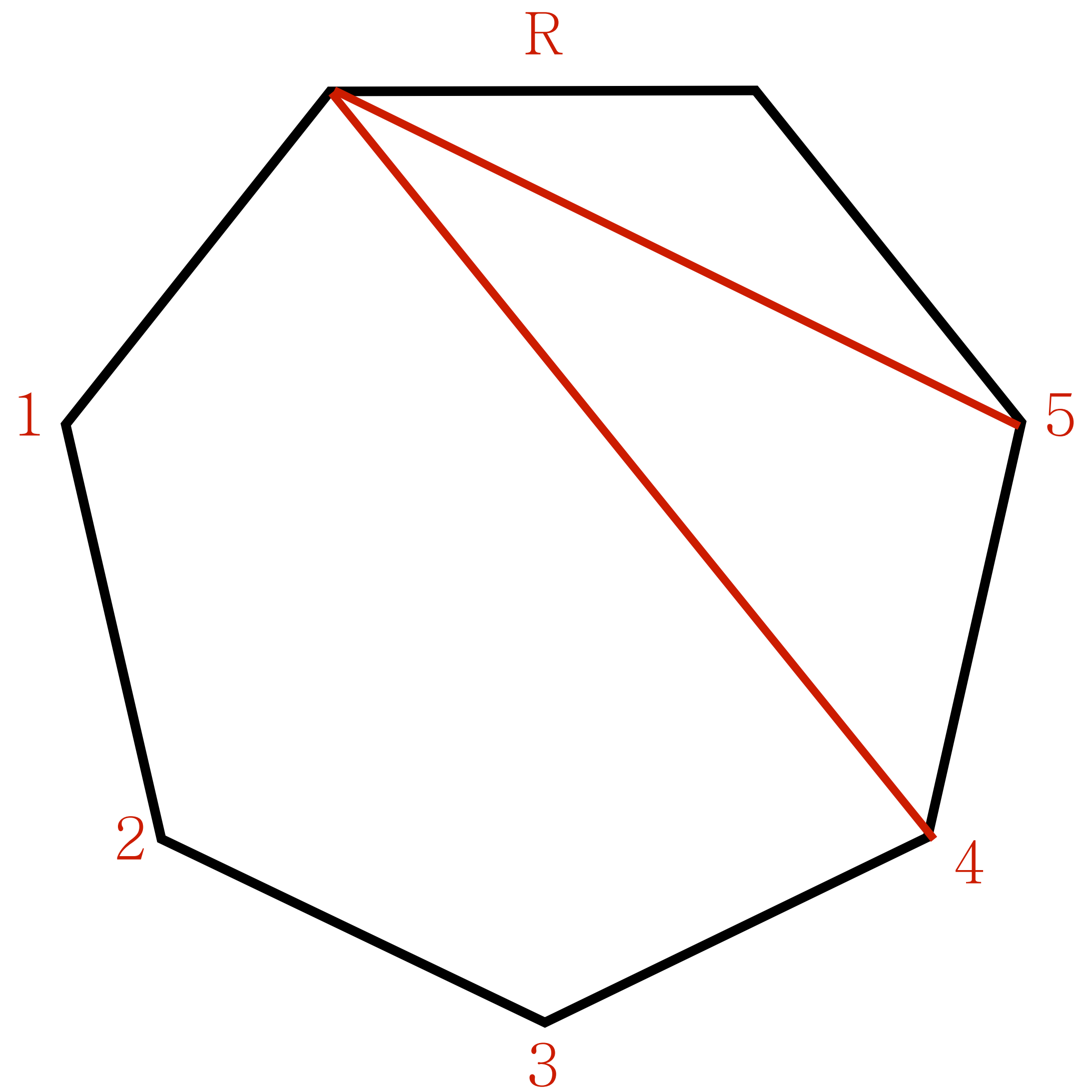
binary tree



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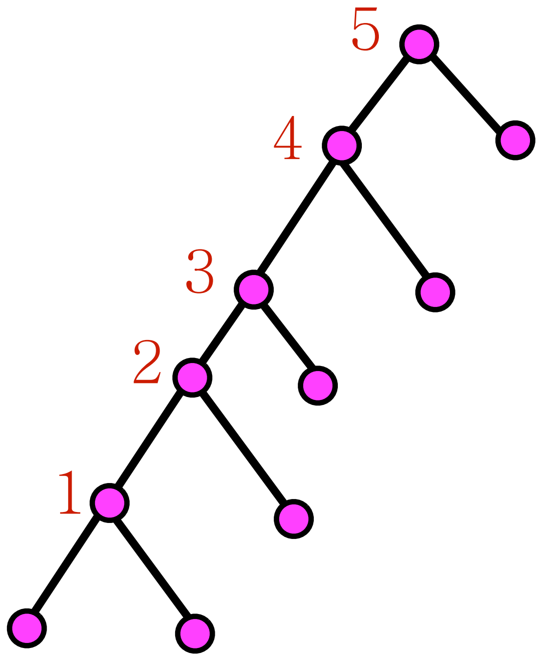
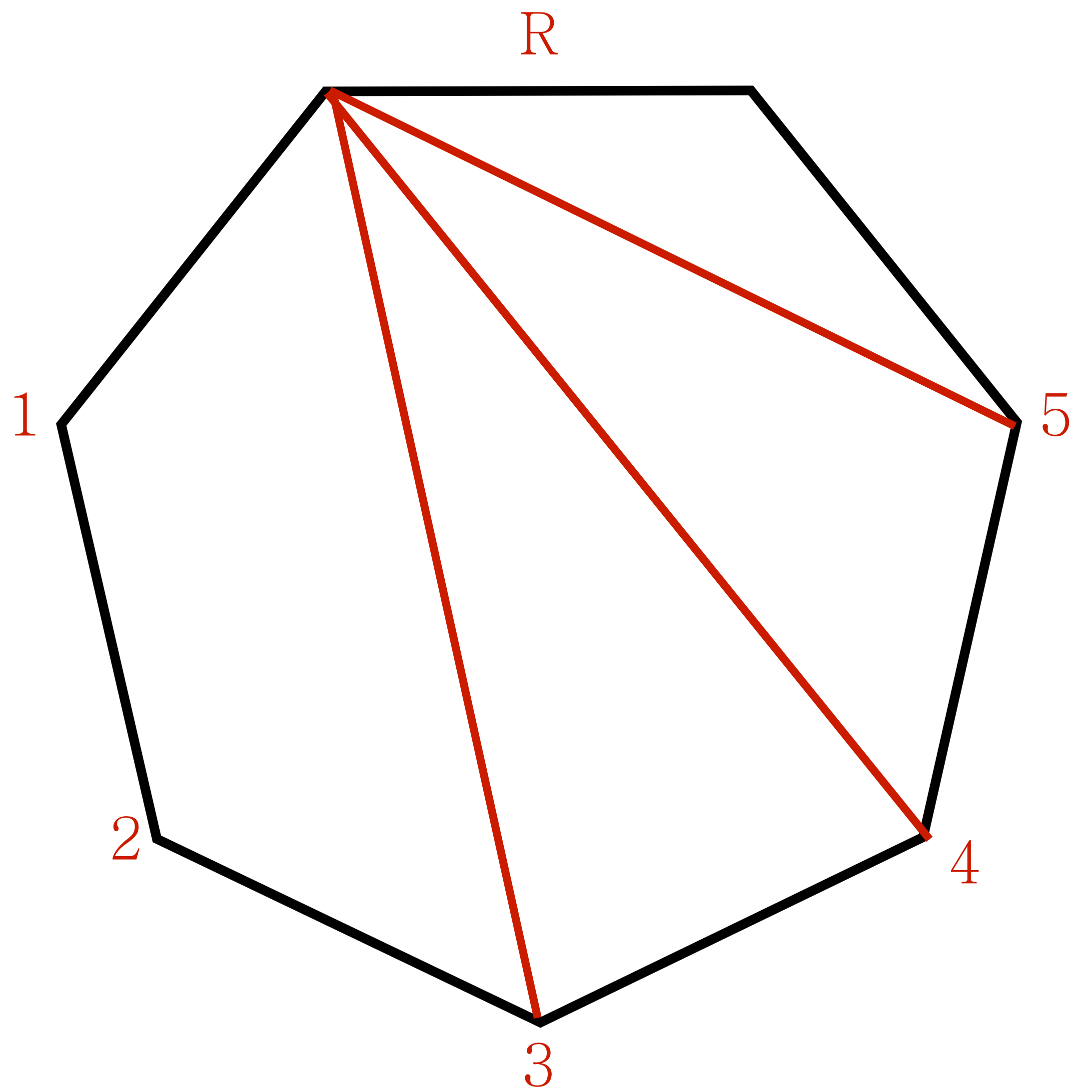
POLYGONS

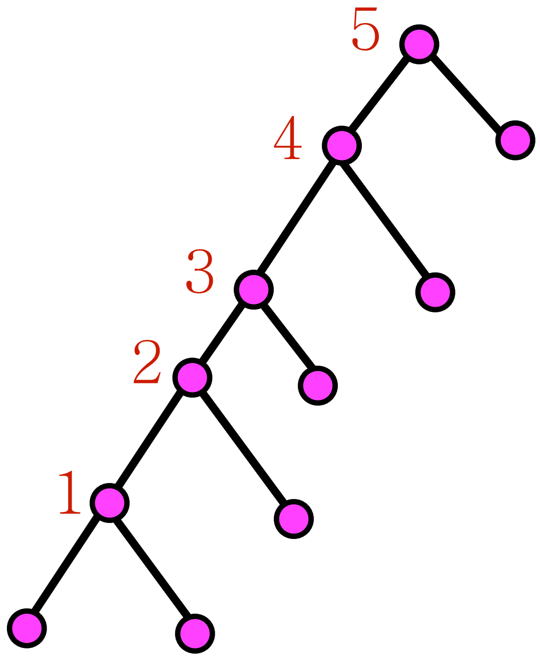


Size n
binary tree



(n+ 2)-gon

POLYGONS



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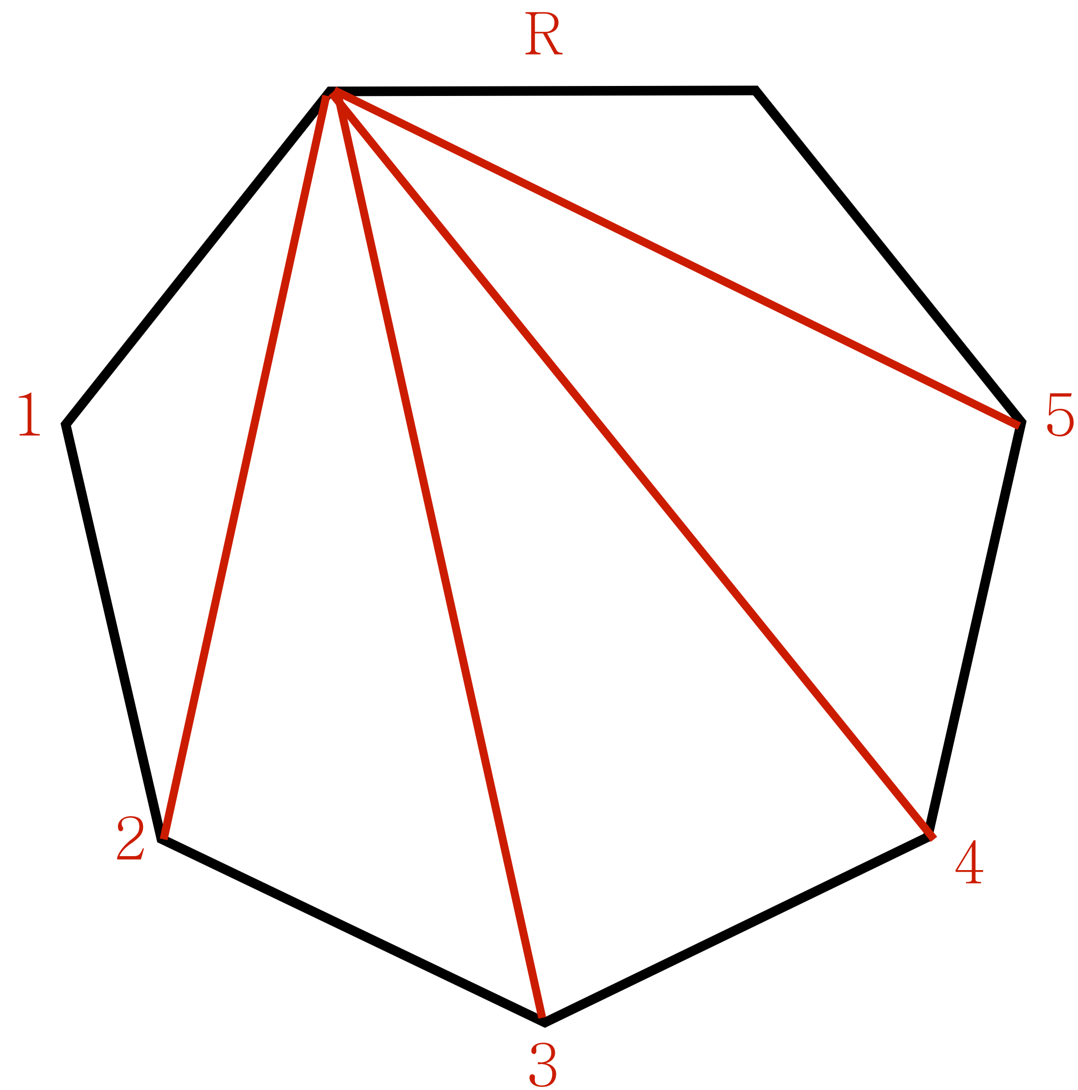
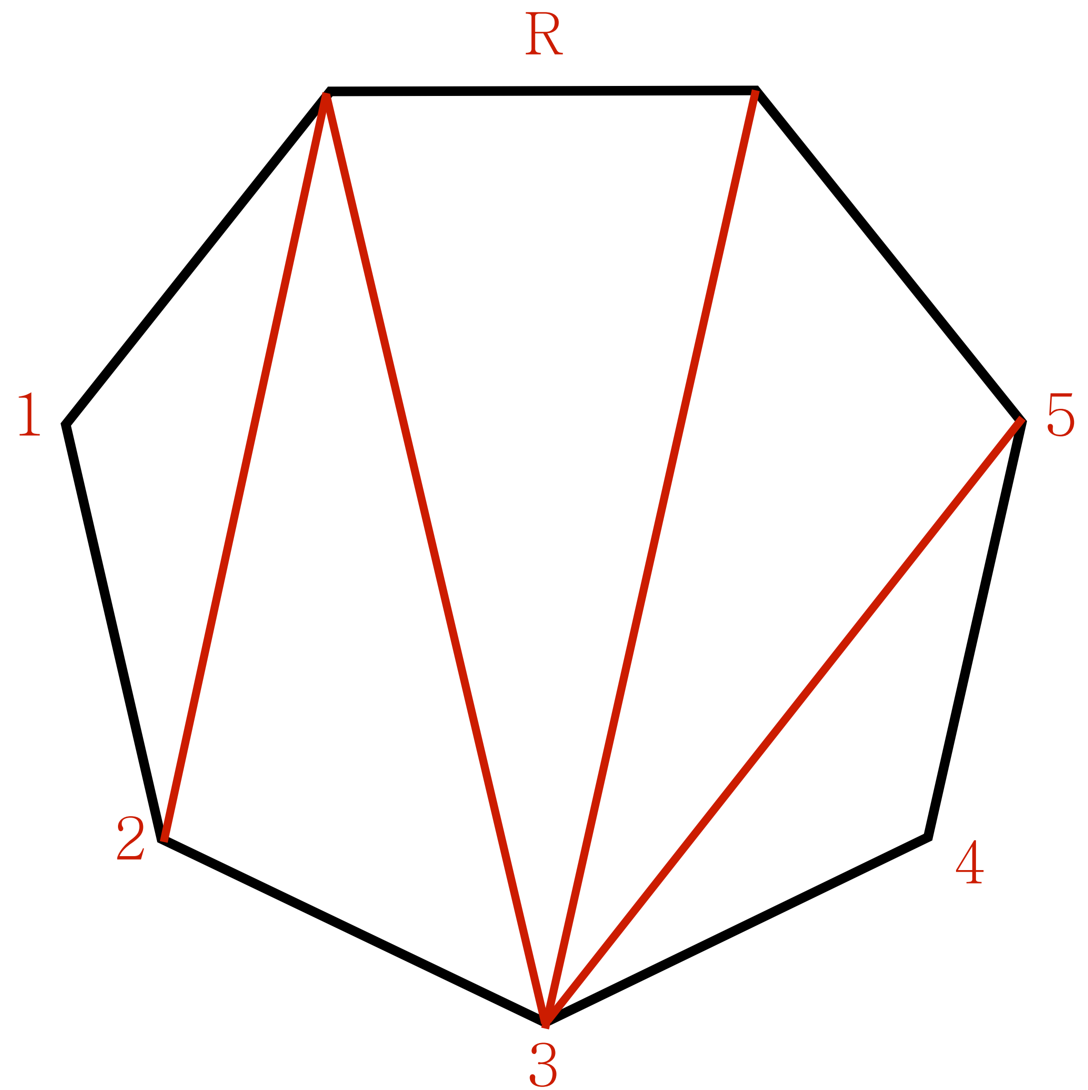
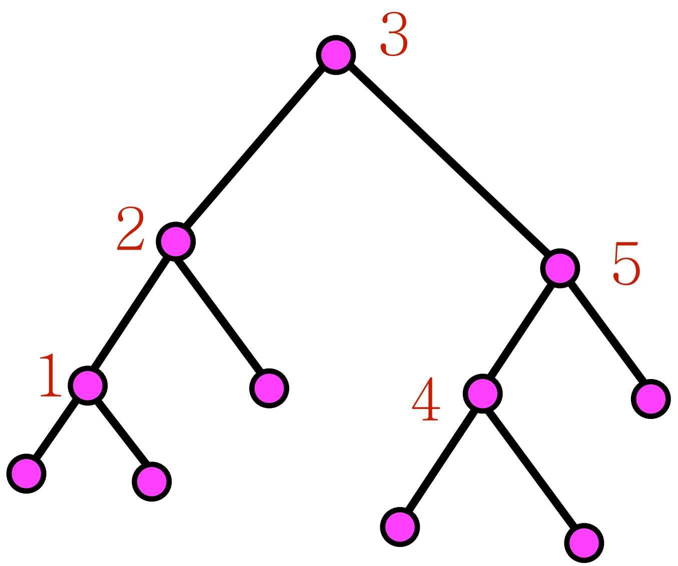


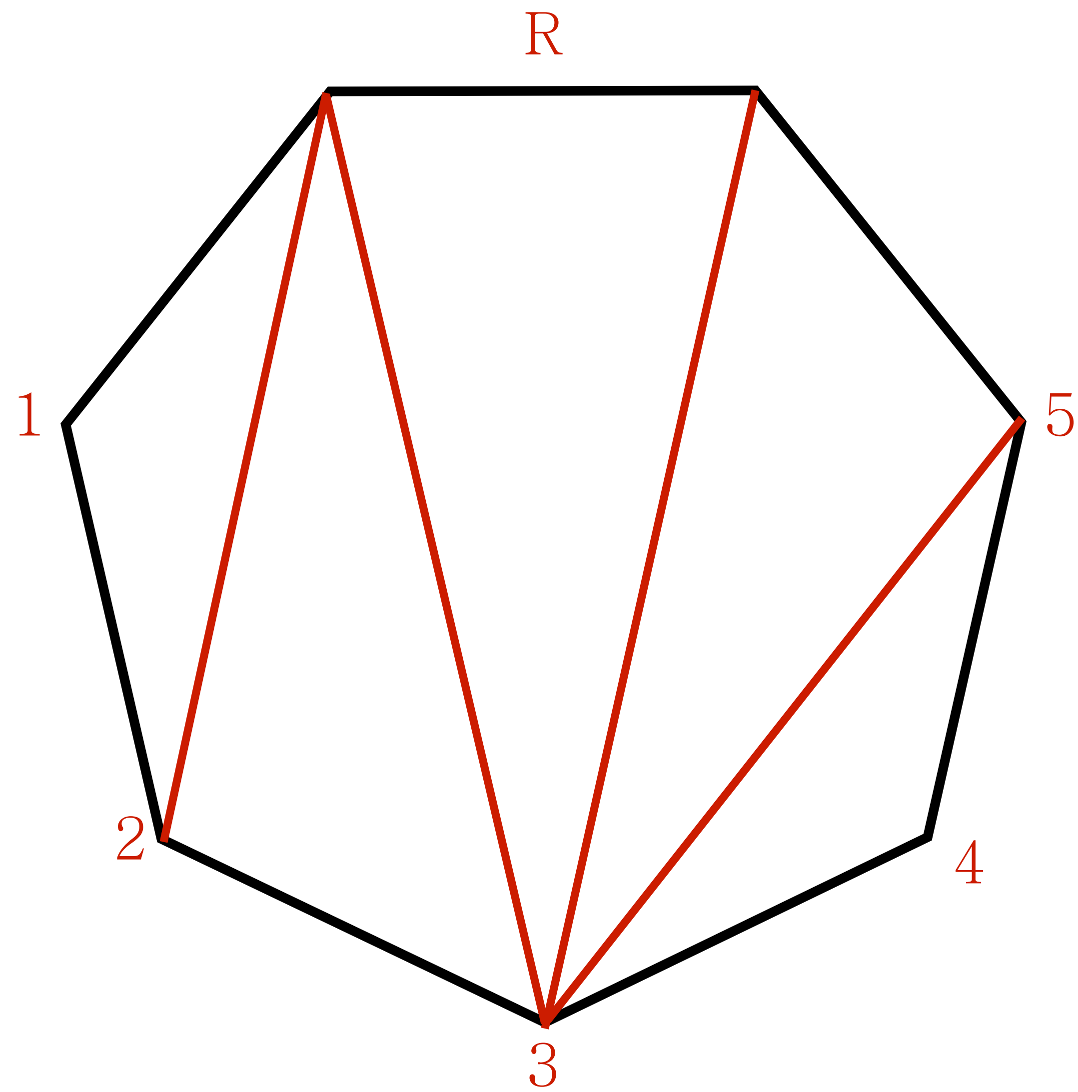
(n+ 2)-gon

POLYGONS

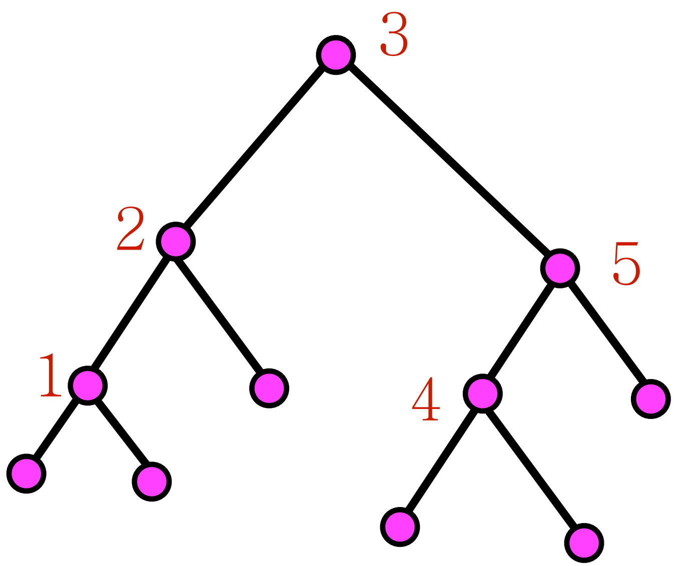


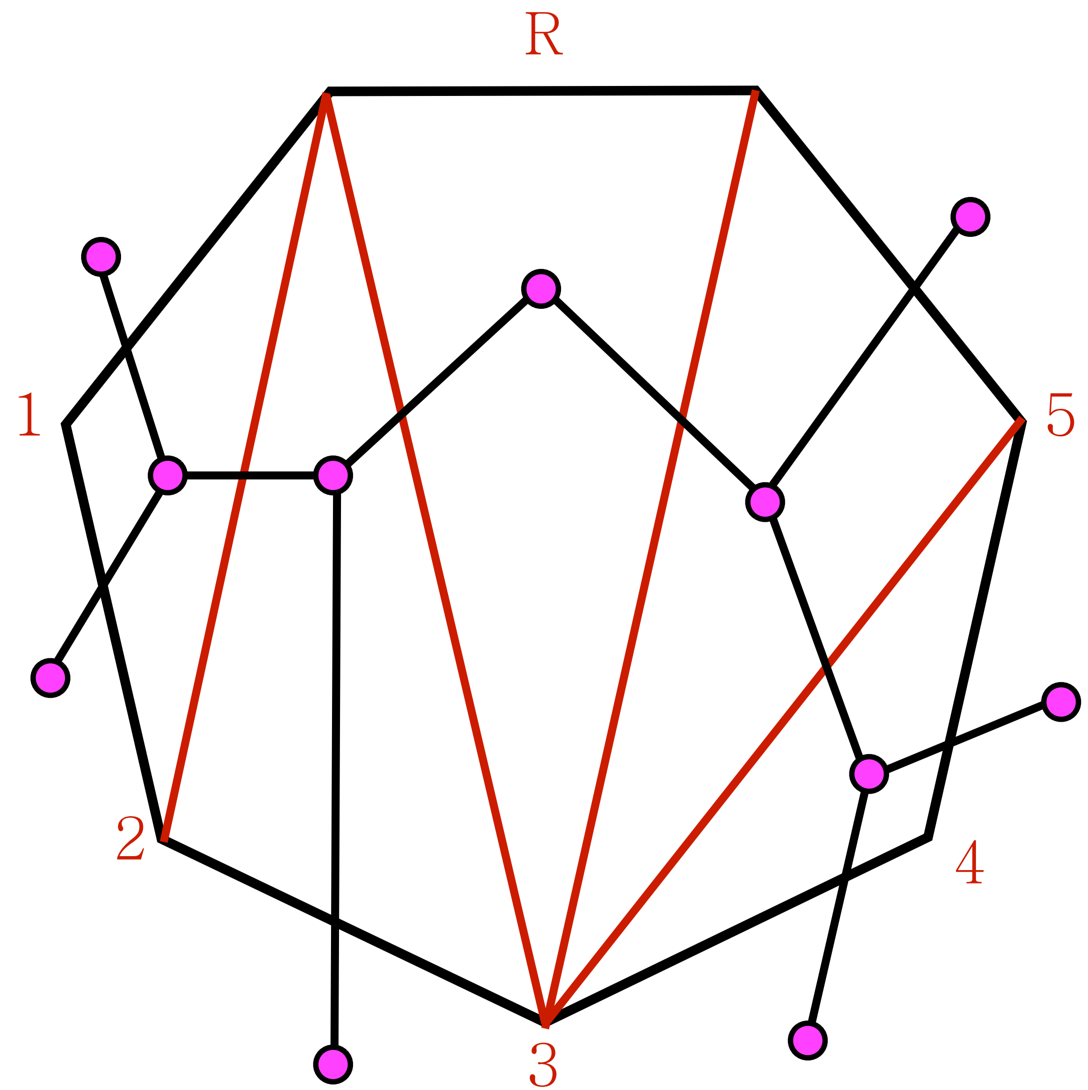
POLYGONS





POLYGONS





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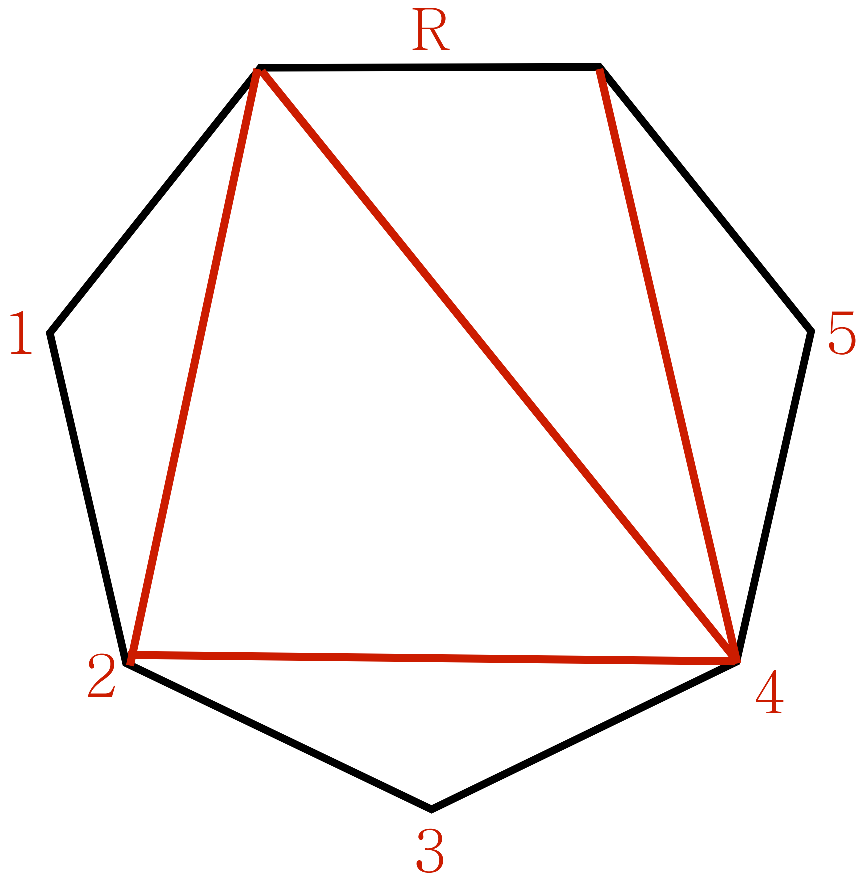
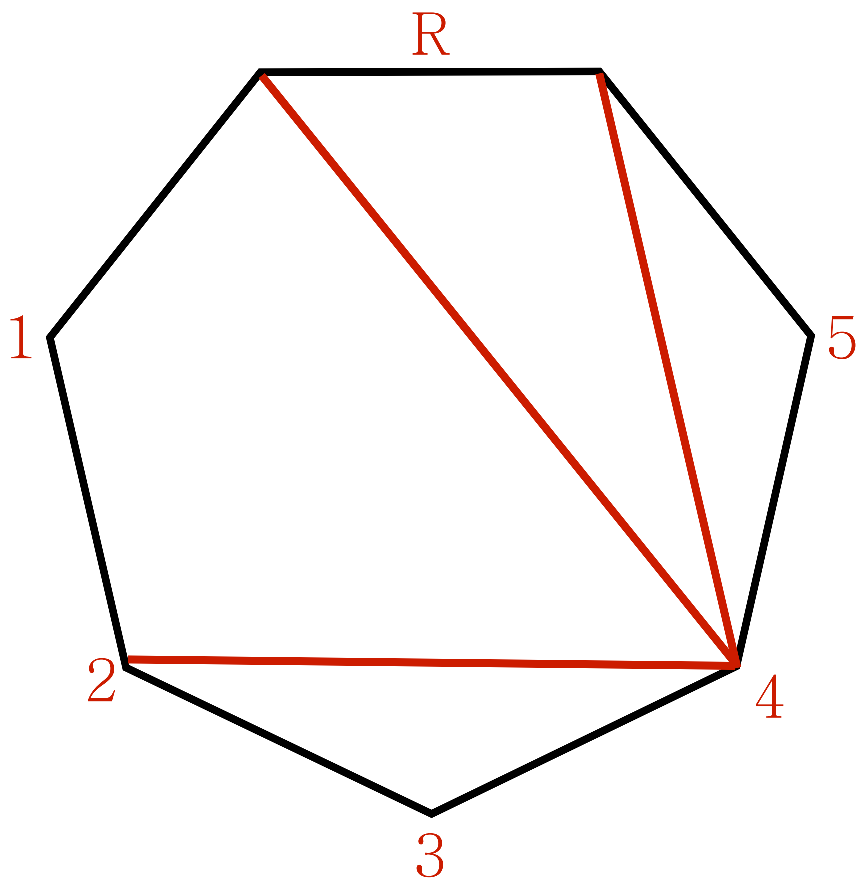


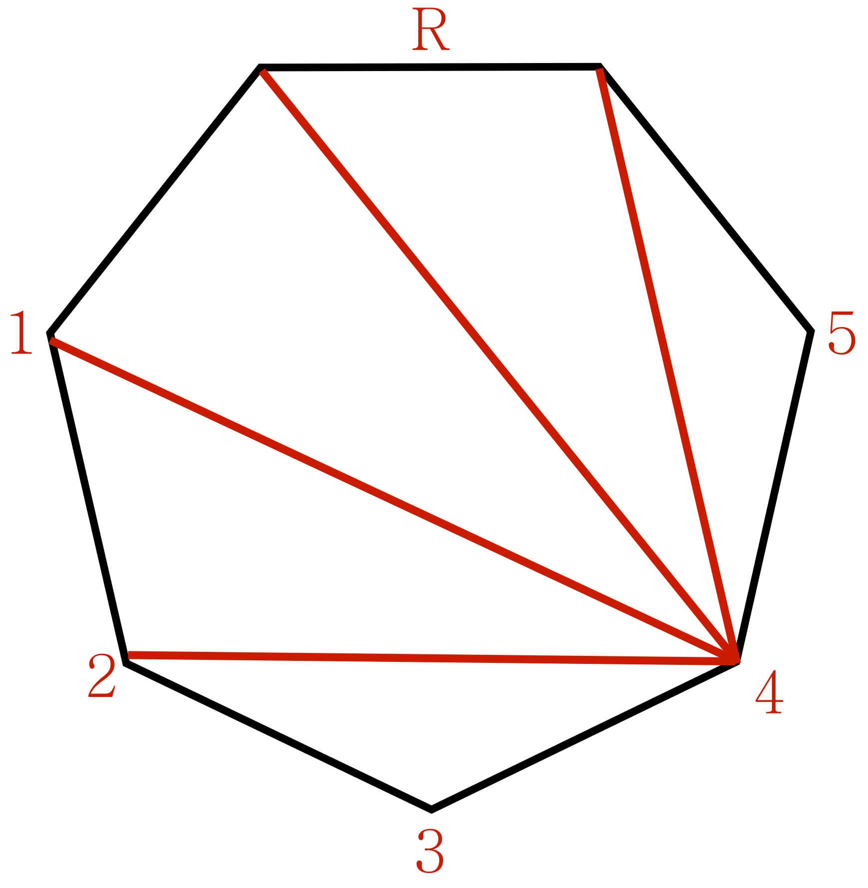
Diagonal Flip

POLYGONS



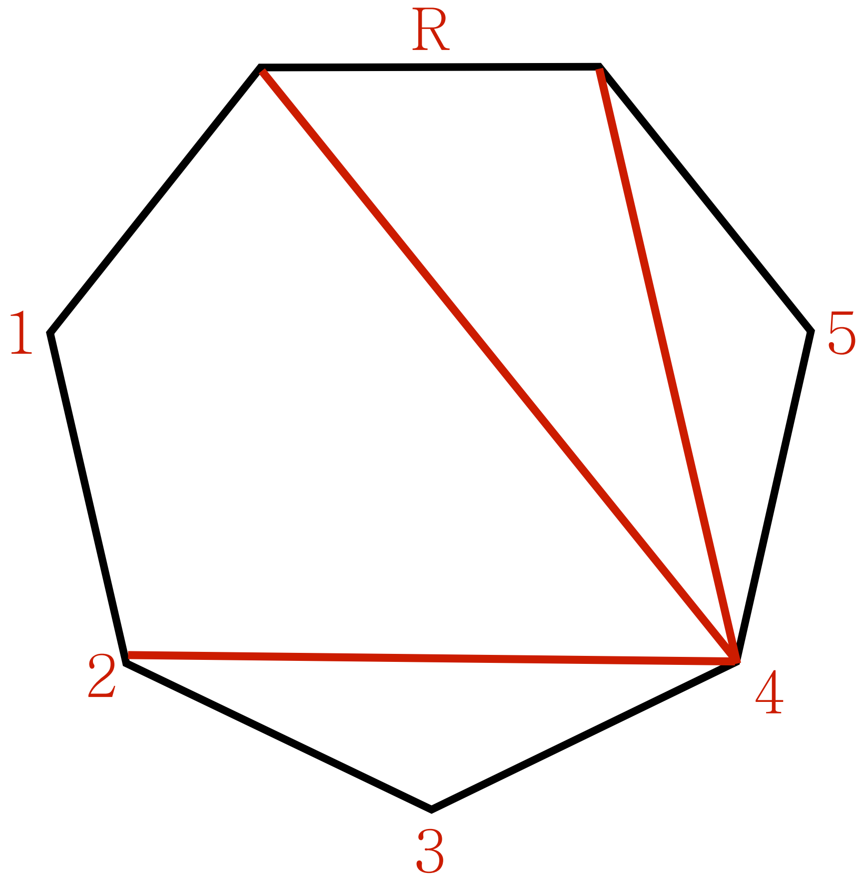
Diagonal Flip

POLYGONS



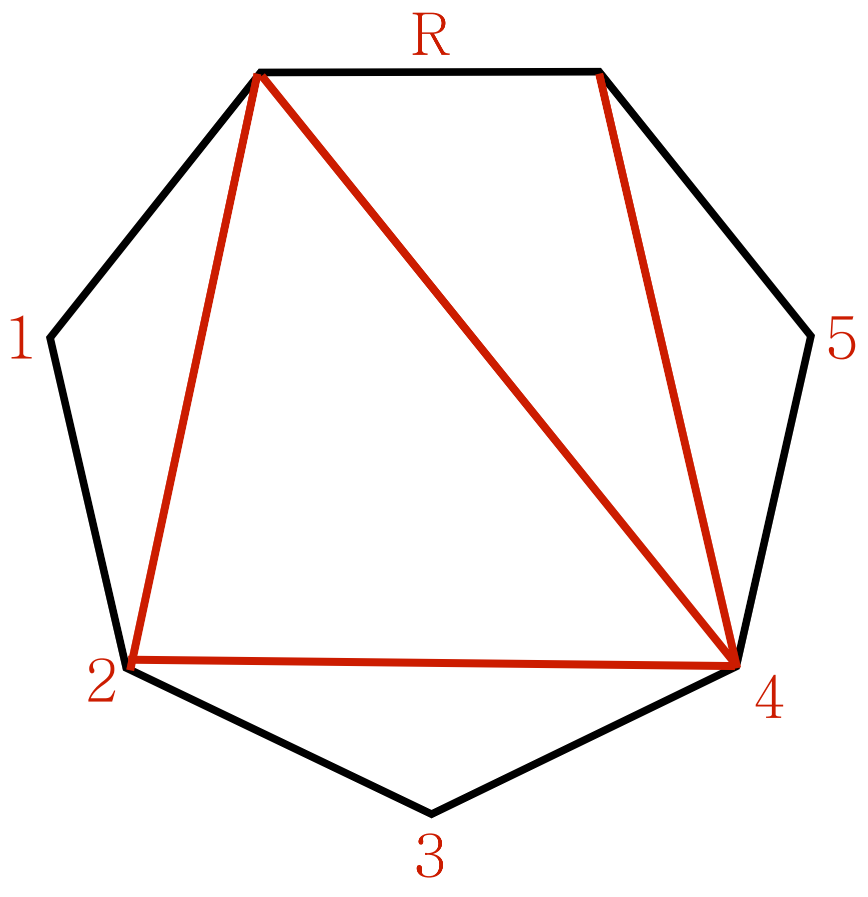
Diagonal Flip

POLYGONS



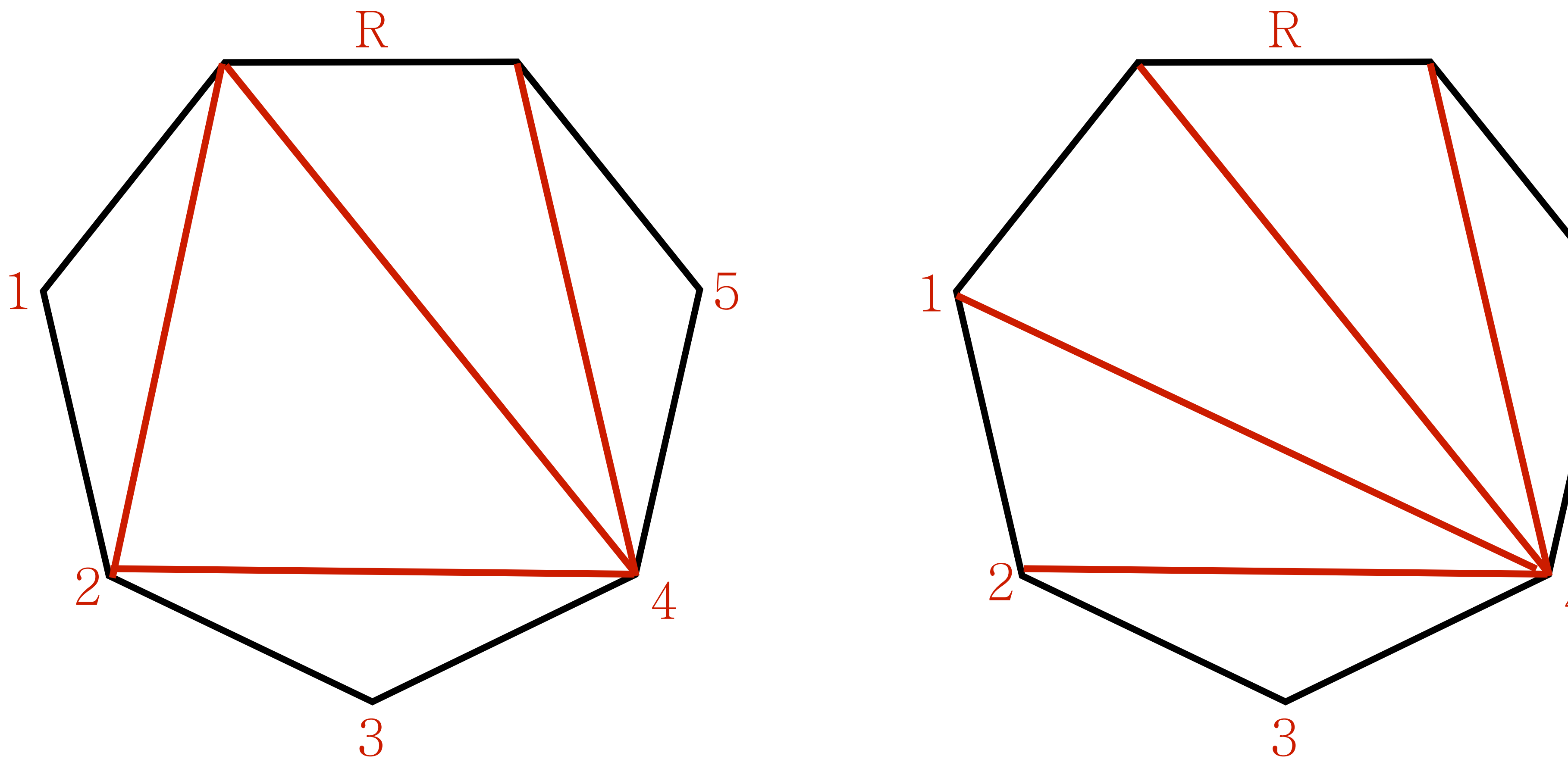
Diagonal Flip

POLYGONS



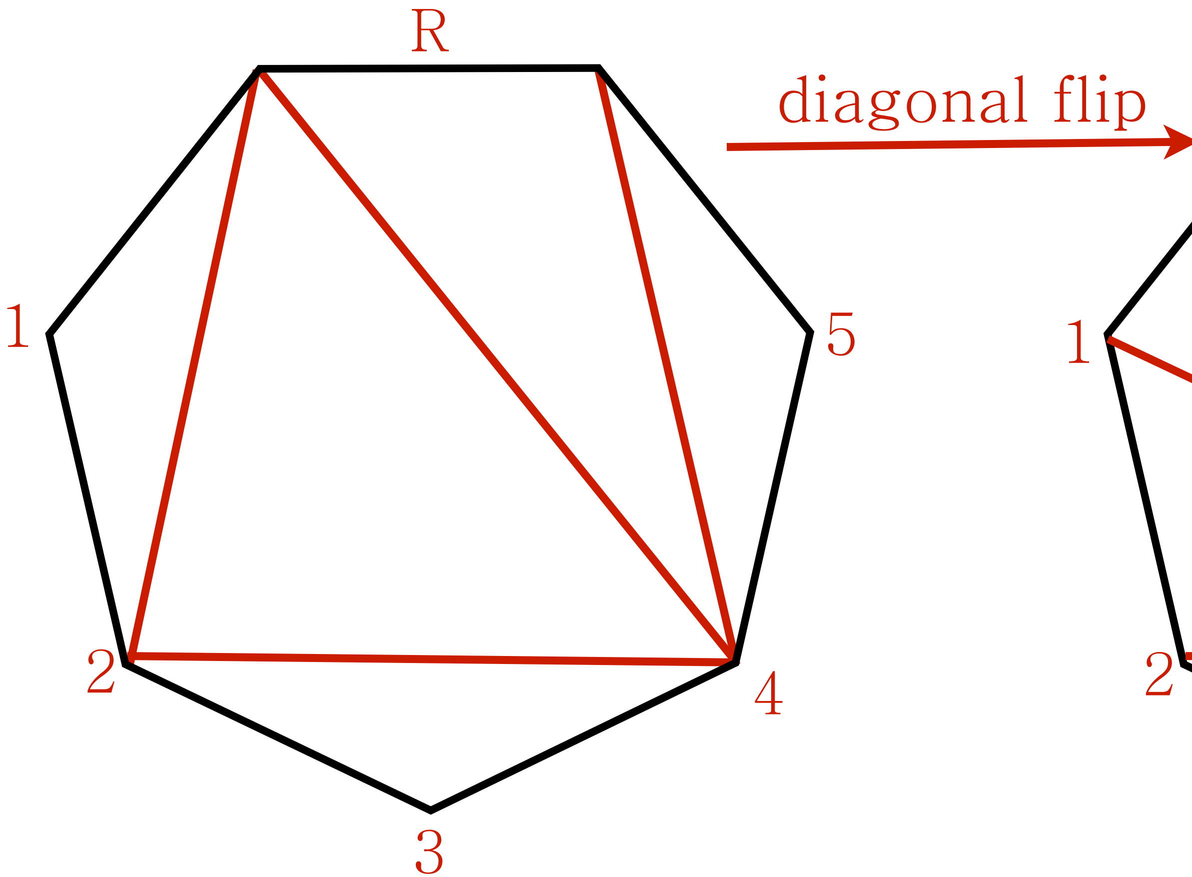
Diagonal Flip

POLYGONS

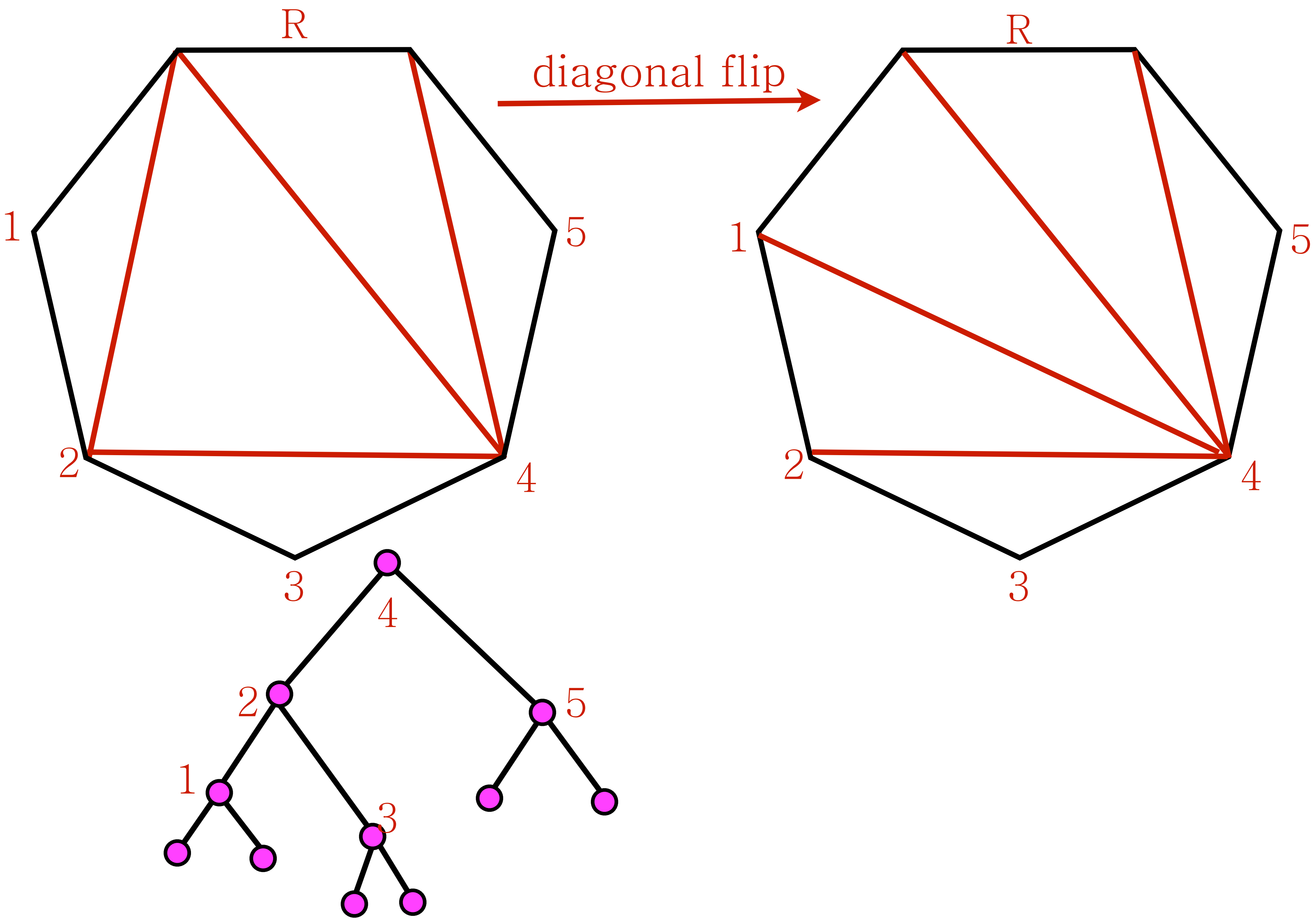


Diagonal Flip

POLYGONS



POLYGONS



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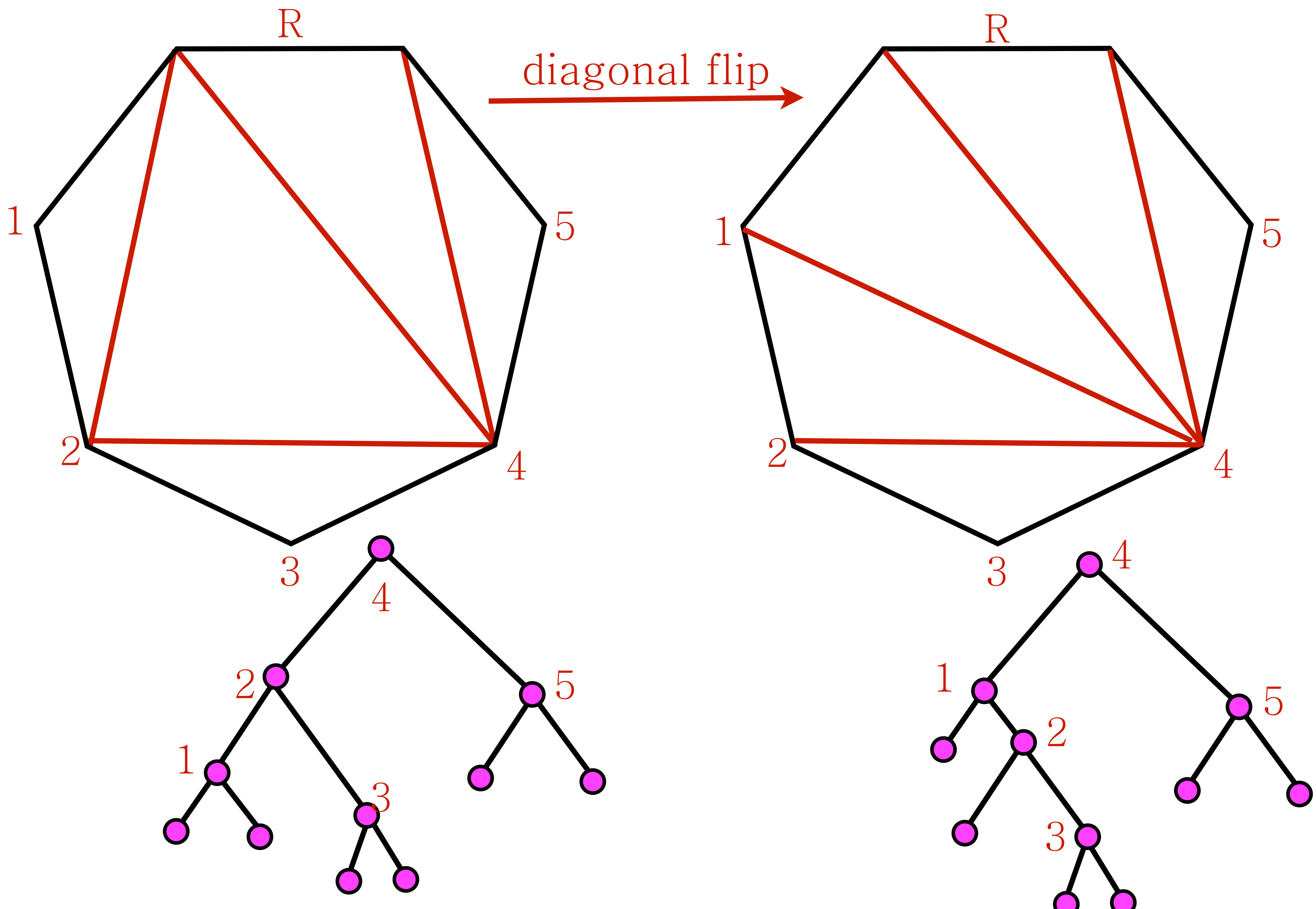
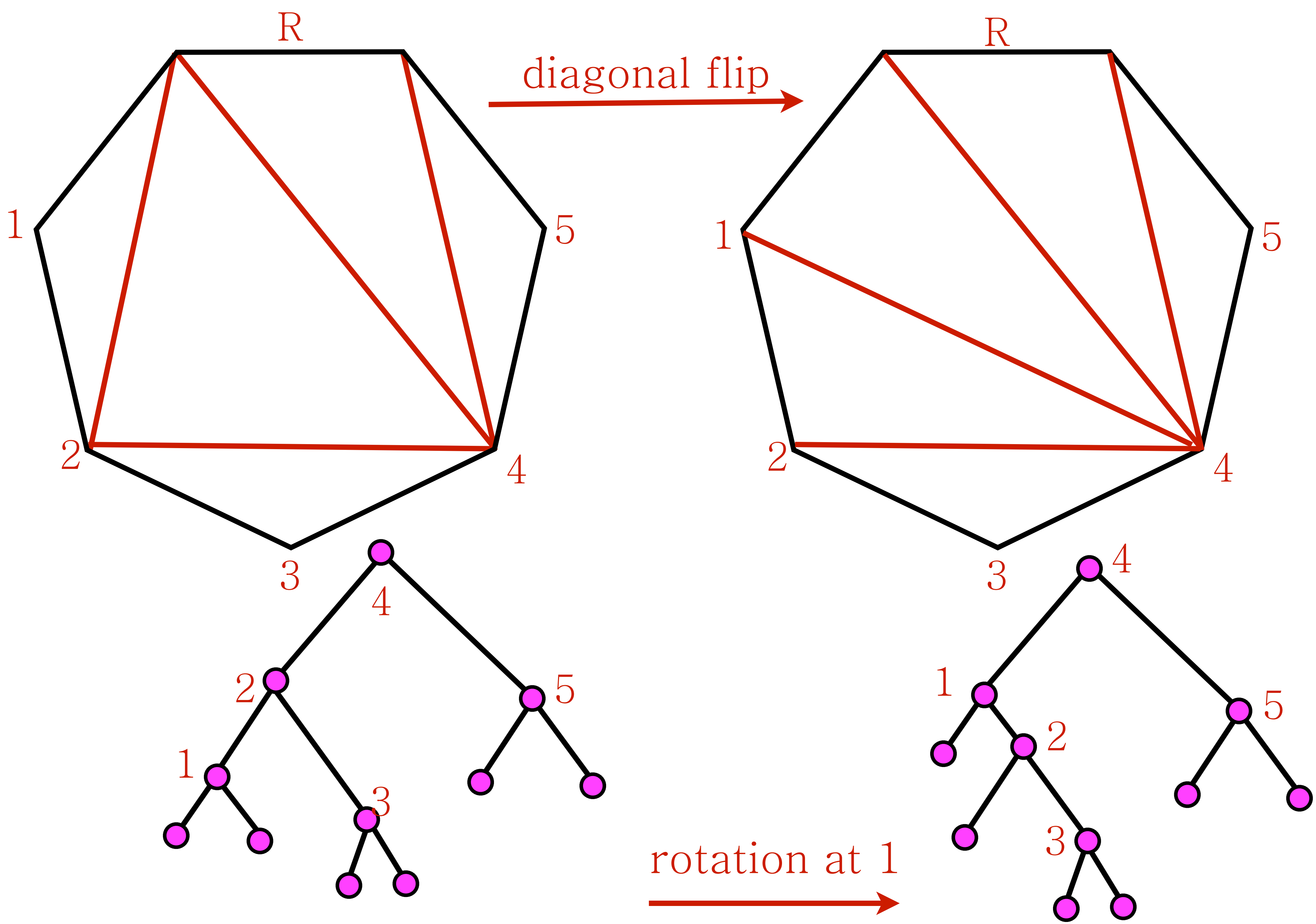
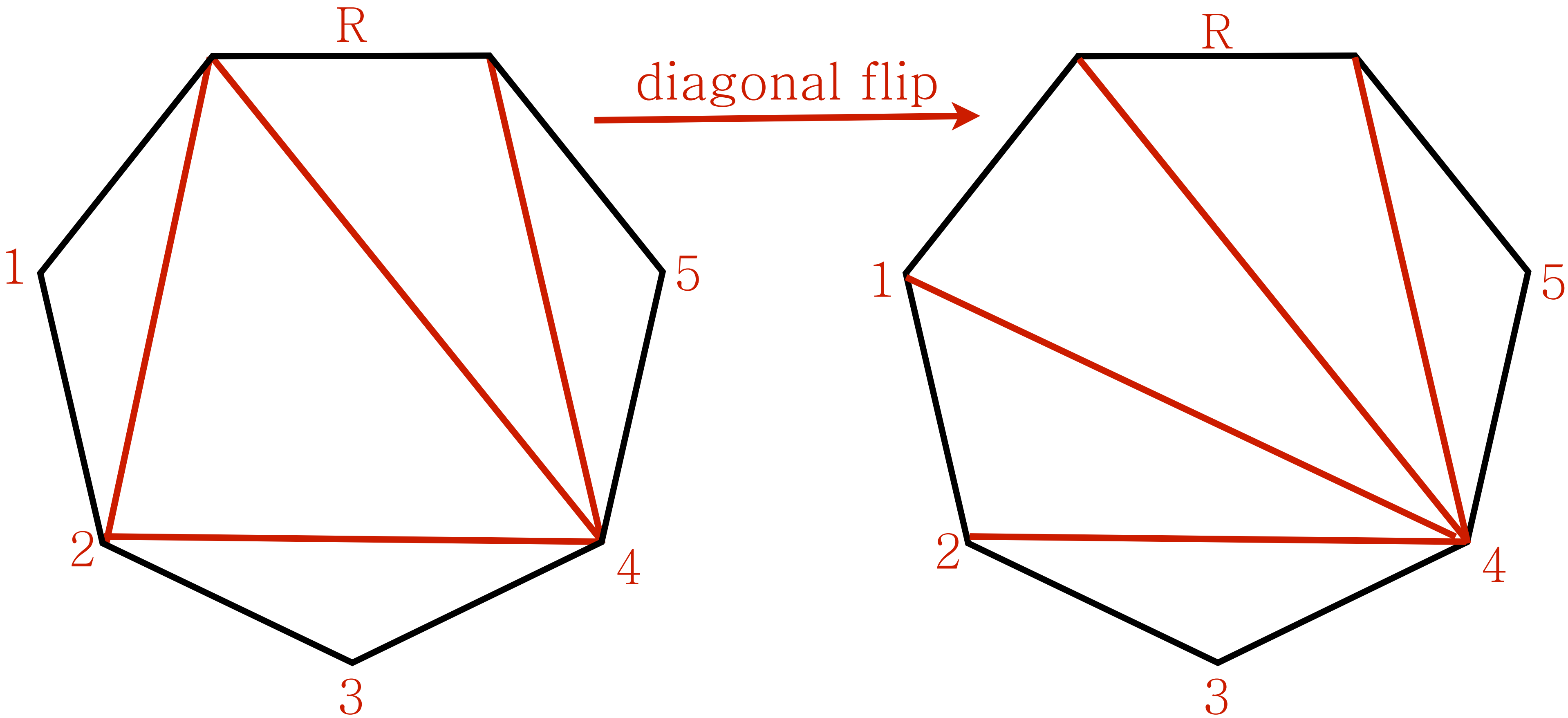


POLYGONS

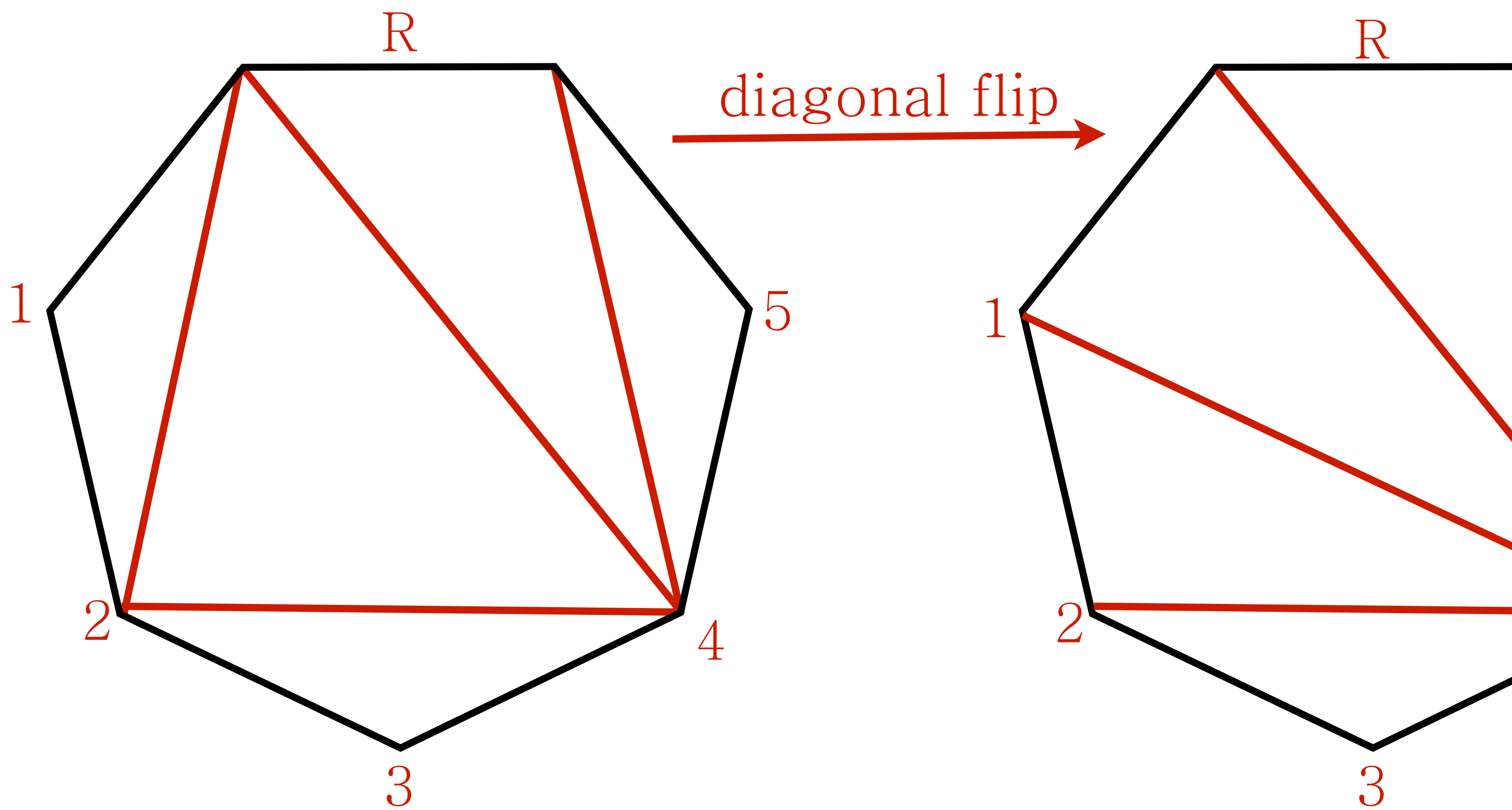


POLYGONS



QUESTION: What is the maximal number of diagonal flips needed to convert between two triangulations of an $(n+2)$-gon?

POLYGONS



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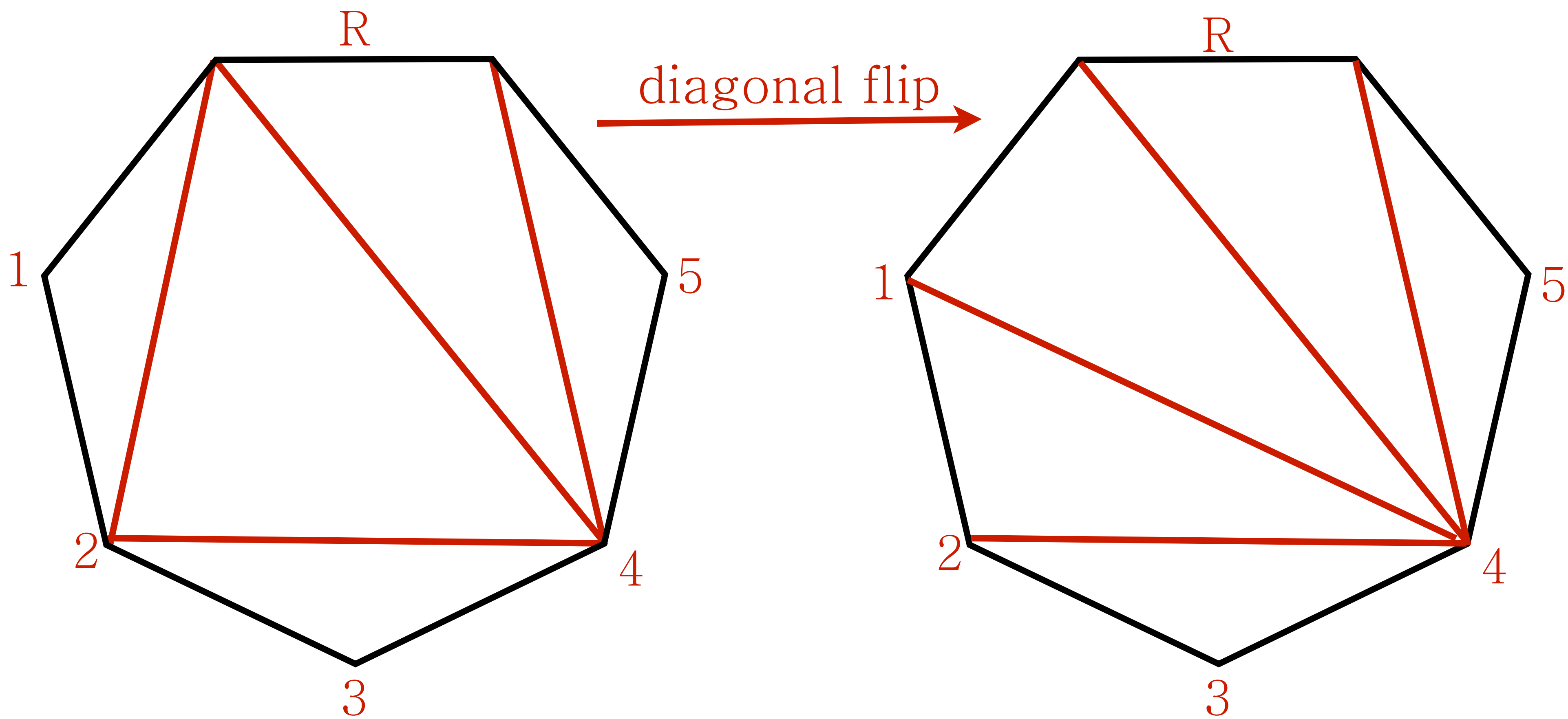
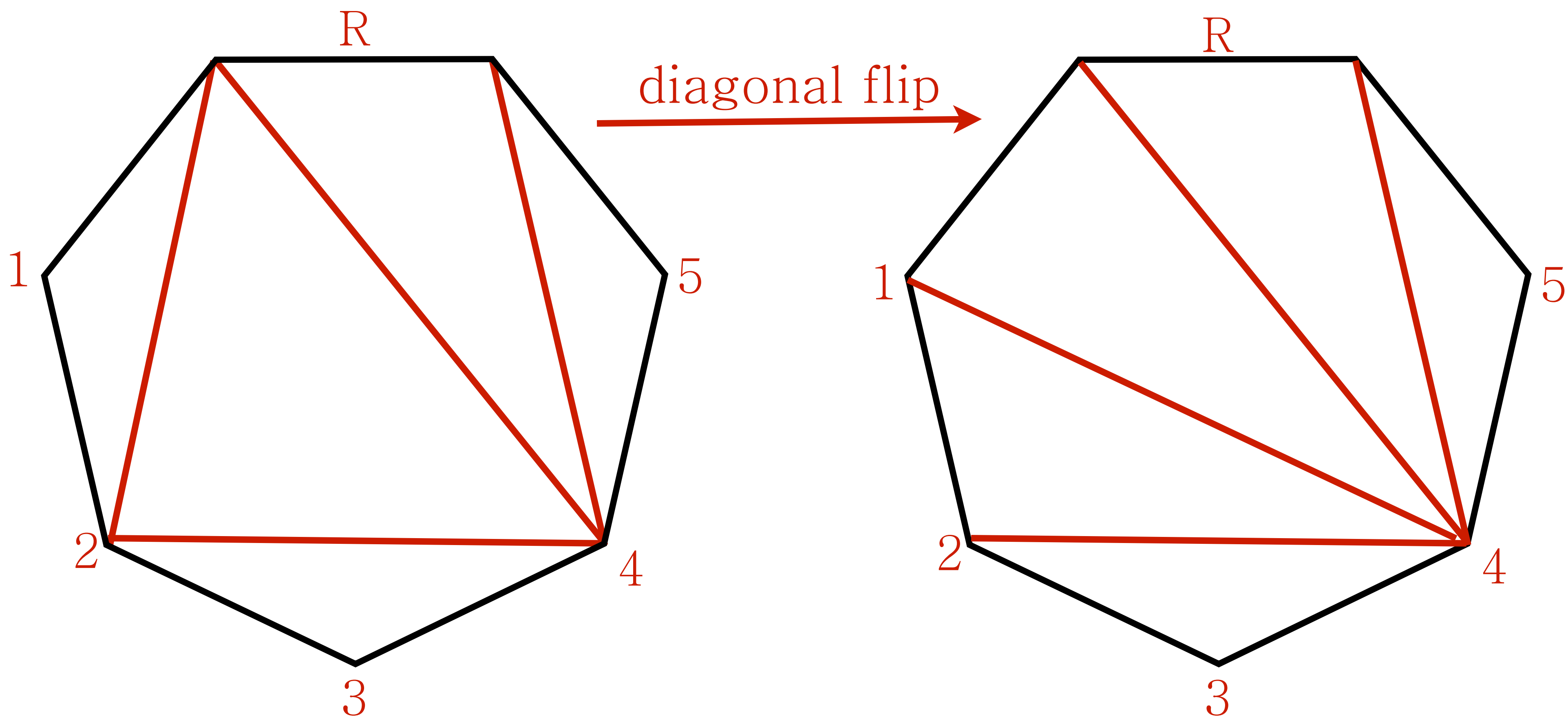
d(n)

POLYGONS



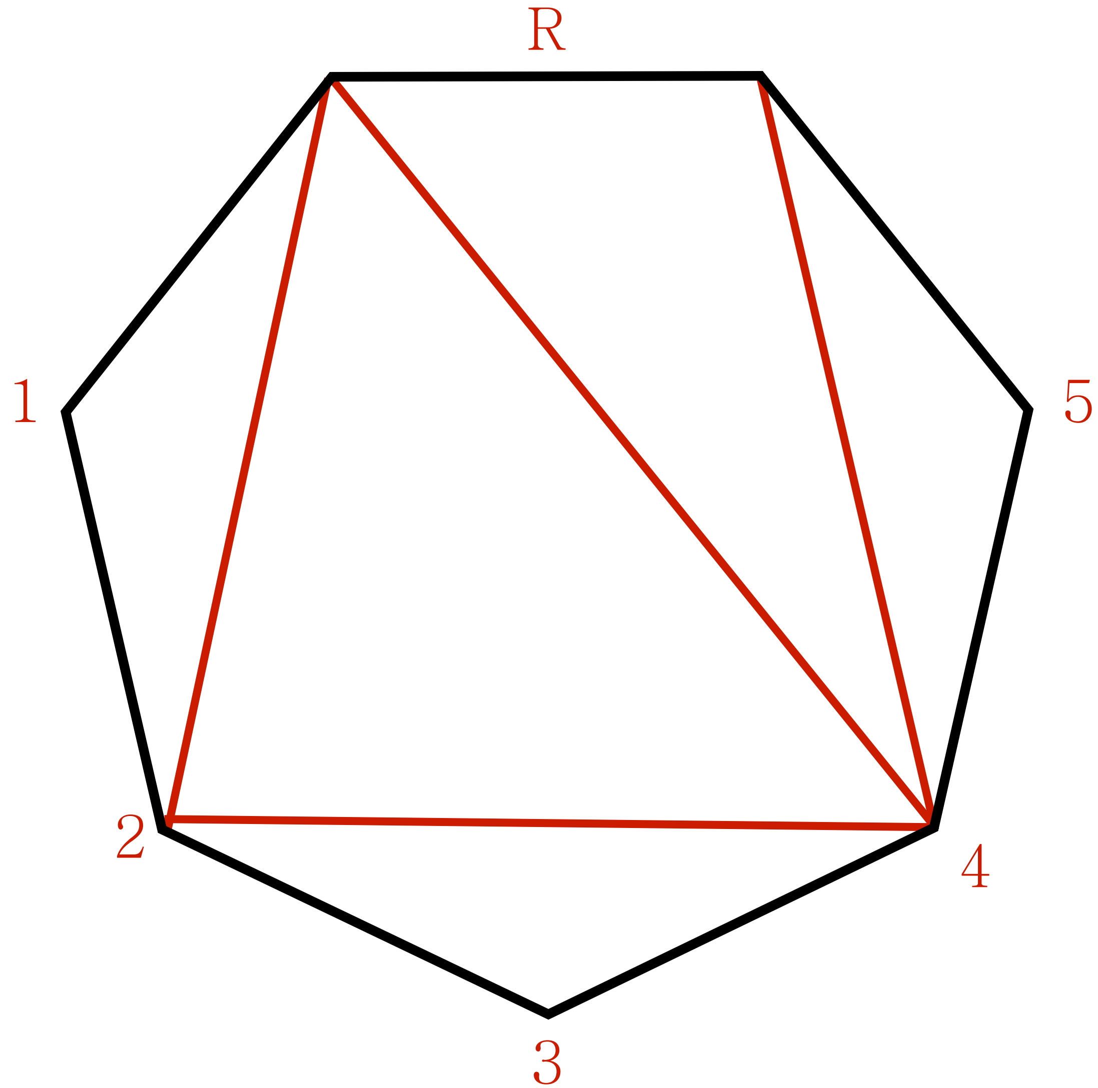
ANSWER:

POLYGONS



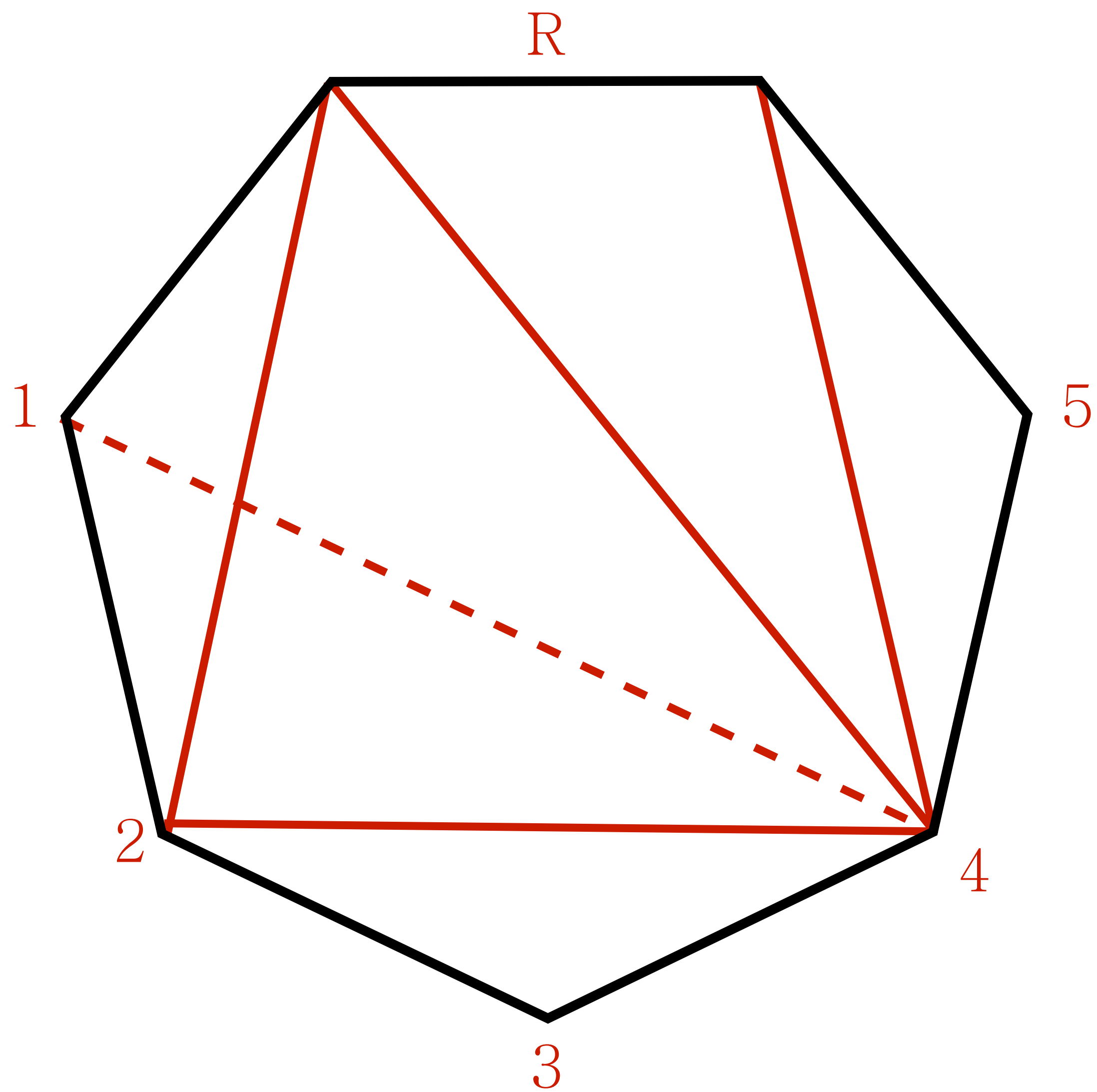
ANSWER: $d(n) \leq 2n - 6$ and equality holds if n large enough.

GEOMETRY



HOW TO PROVE: $d(n) \geq 2n - 6$ if n large enough.

GEOMETRY



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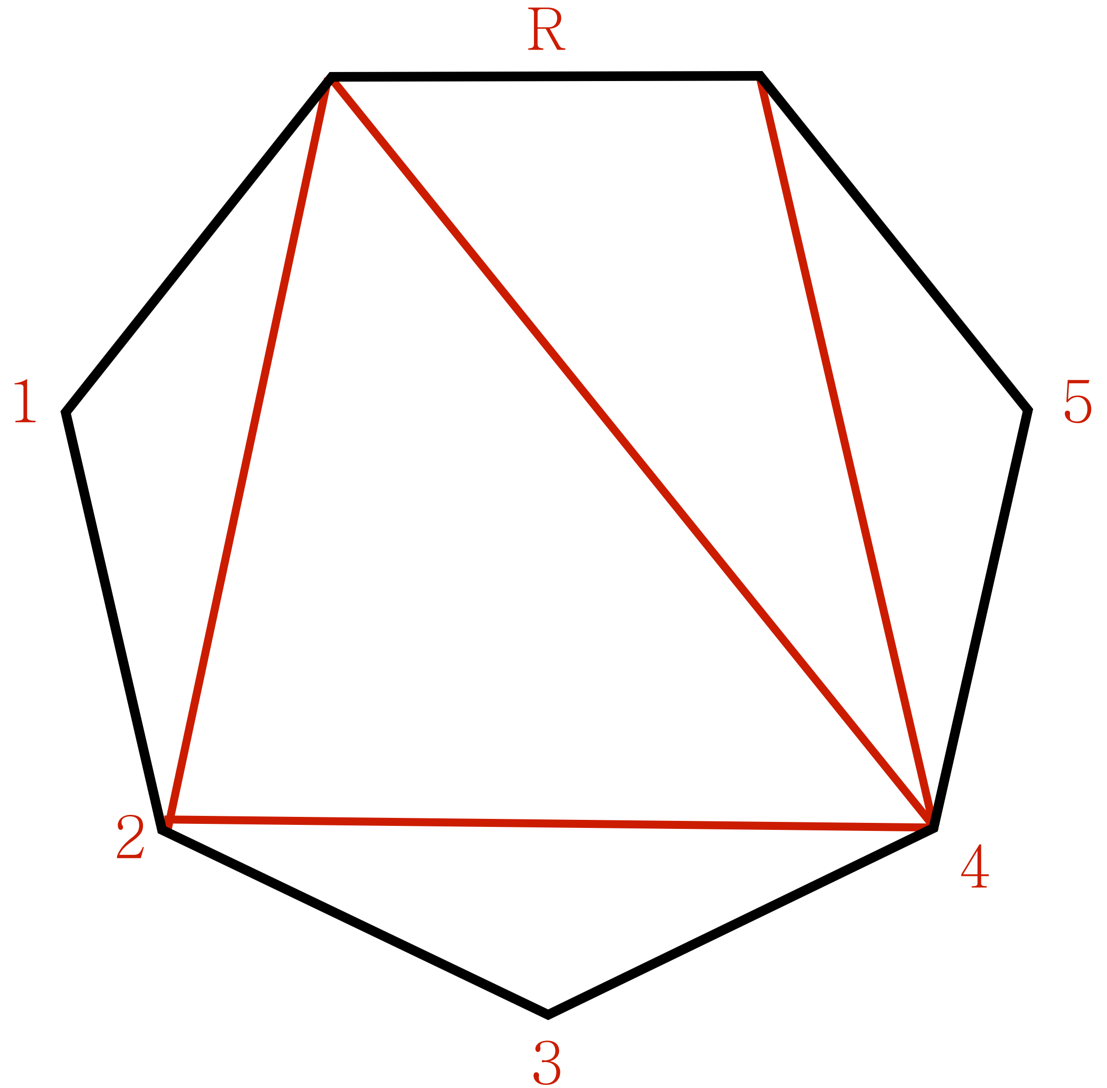

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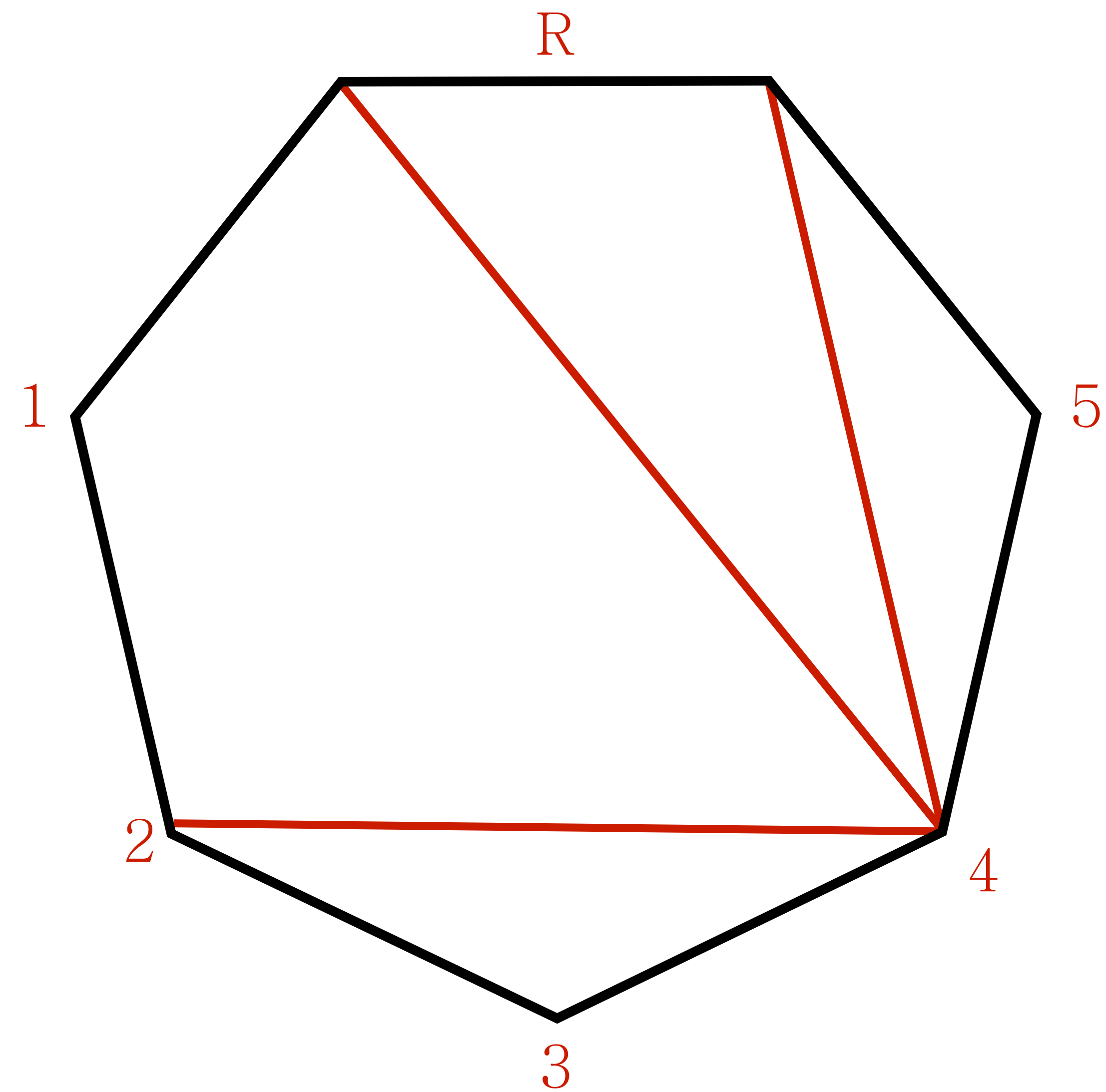

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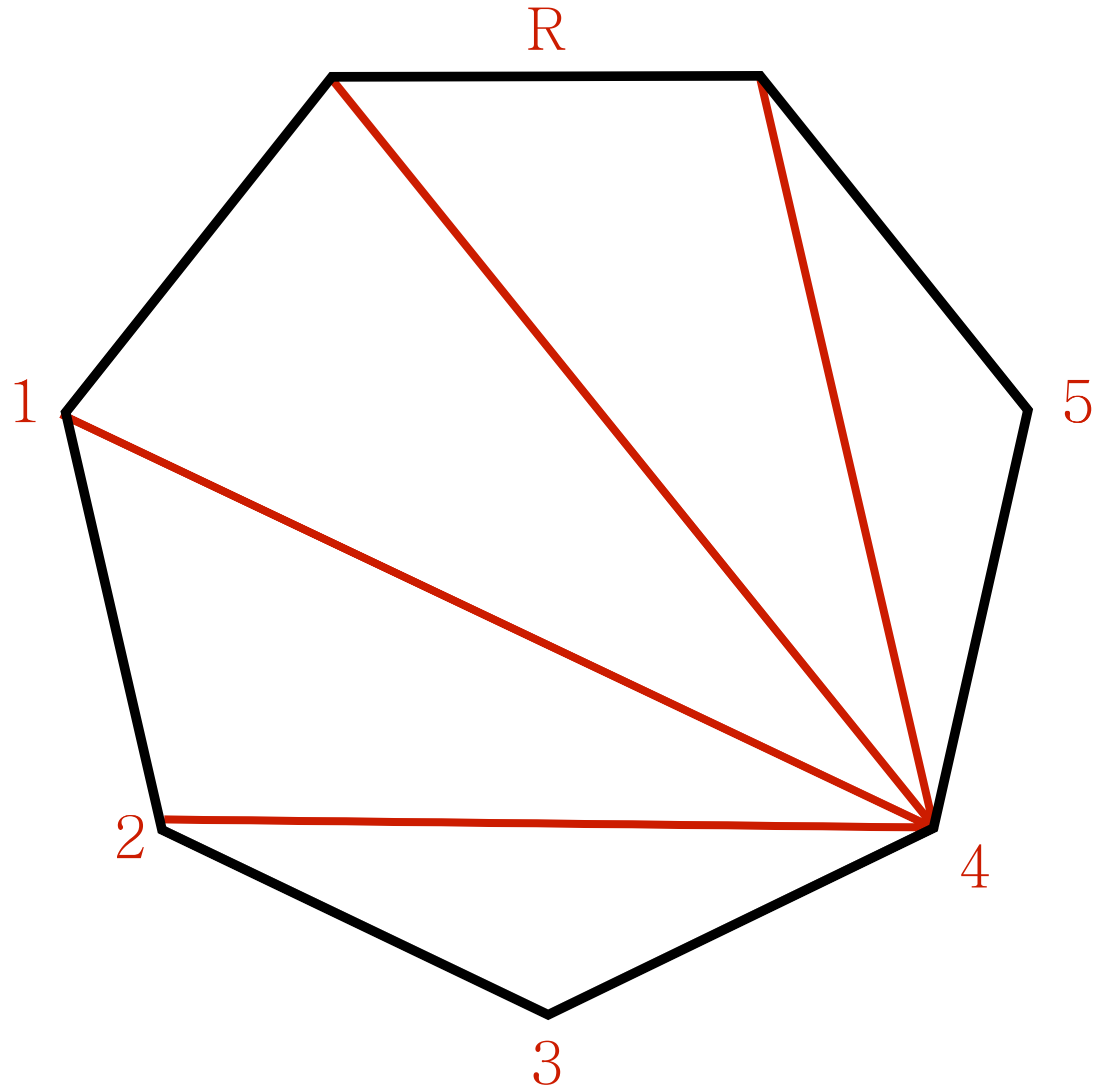

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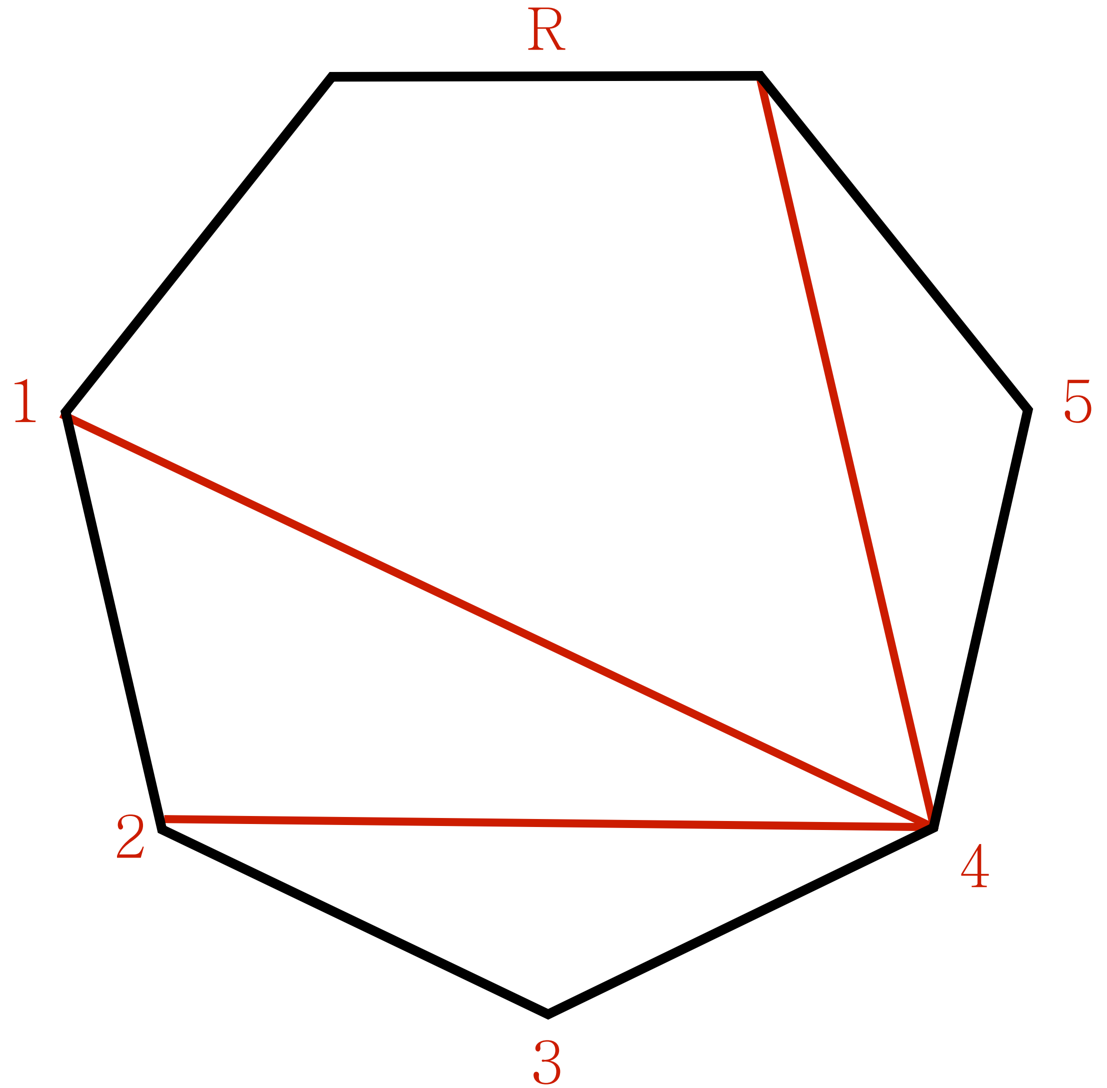

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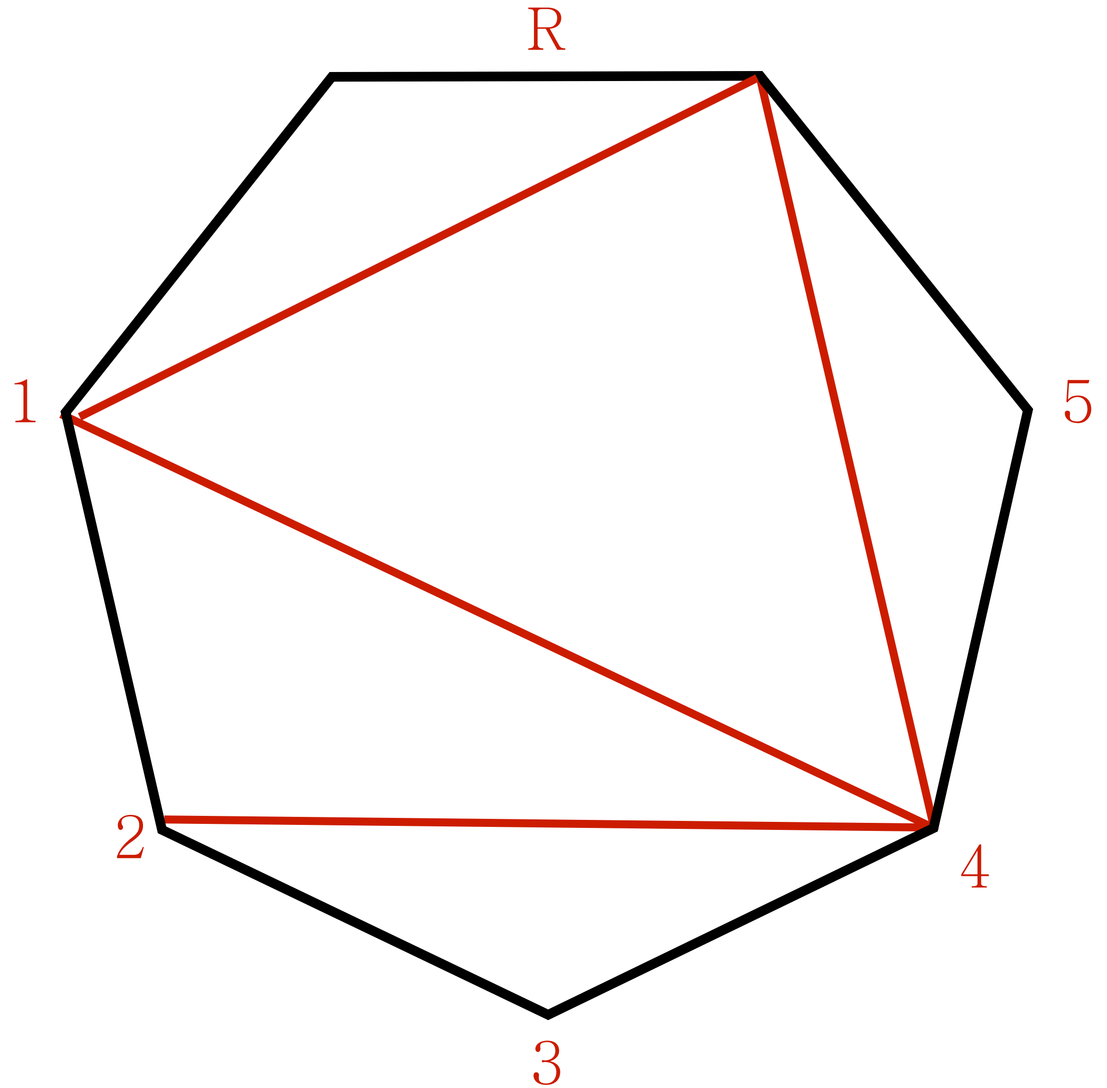

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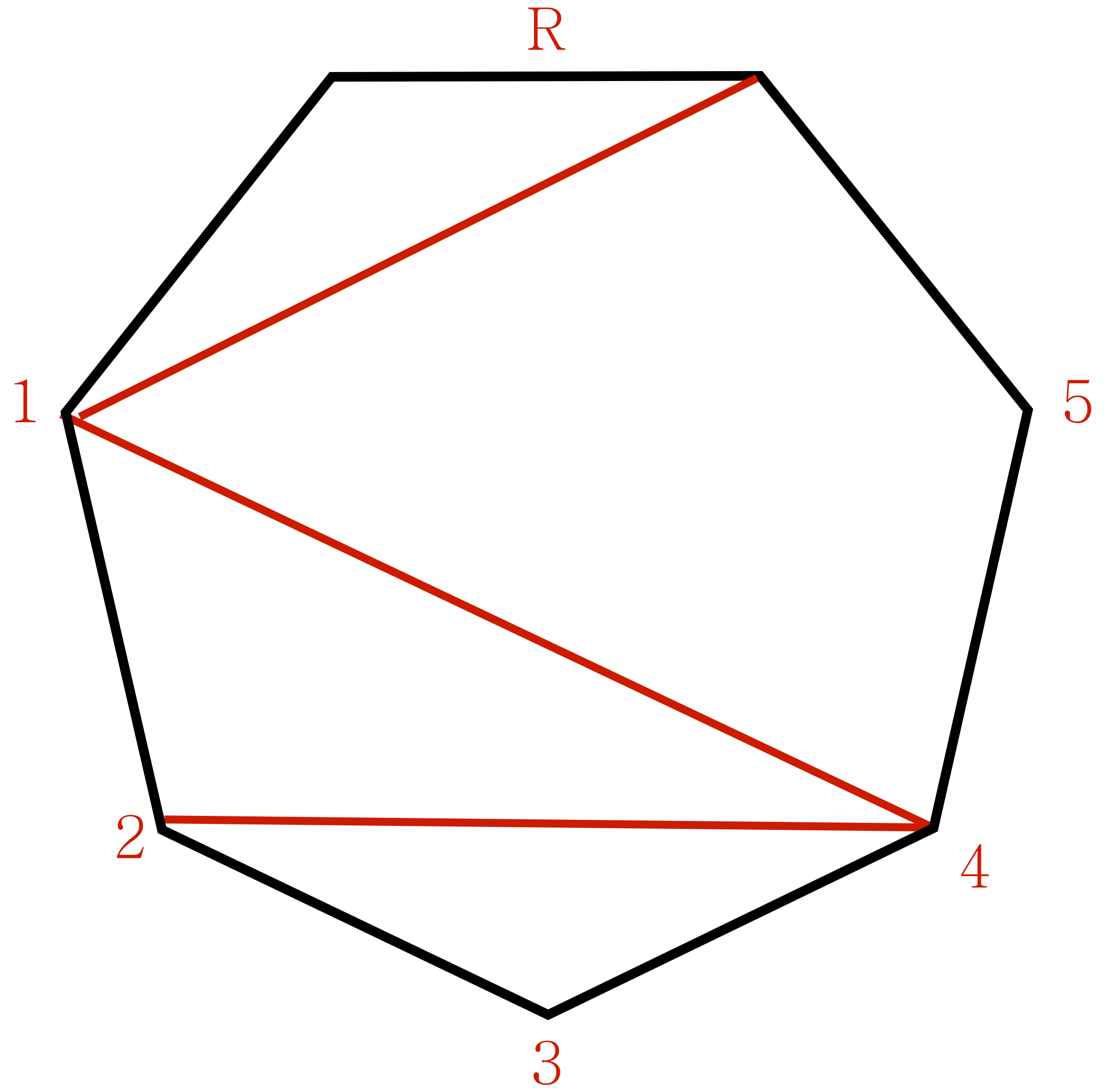

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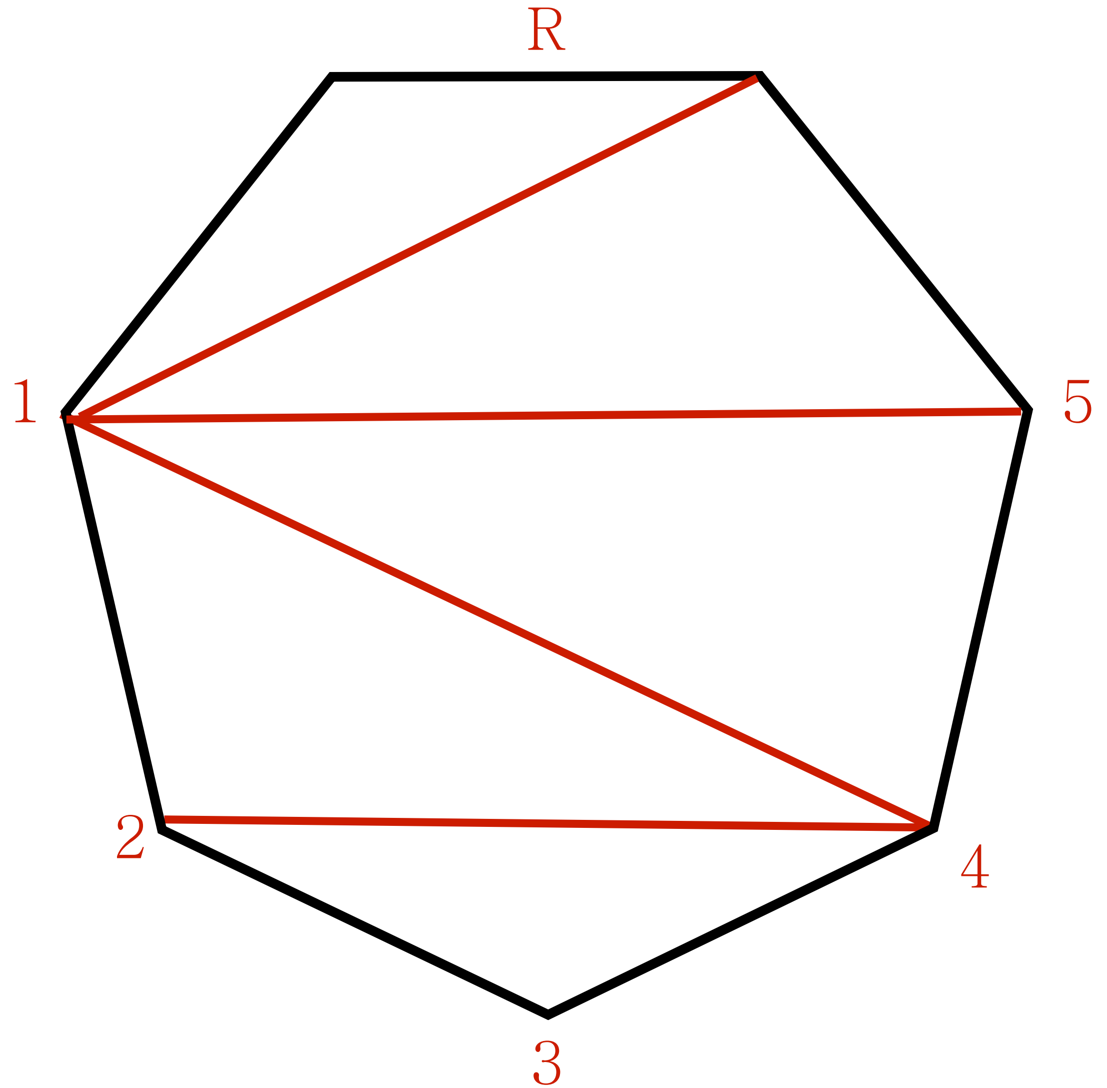

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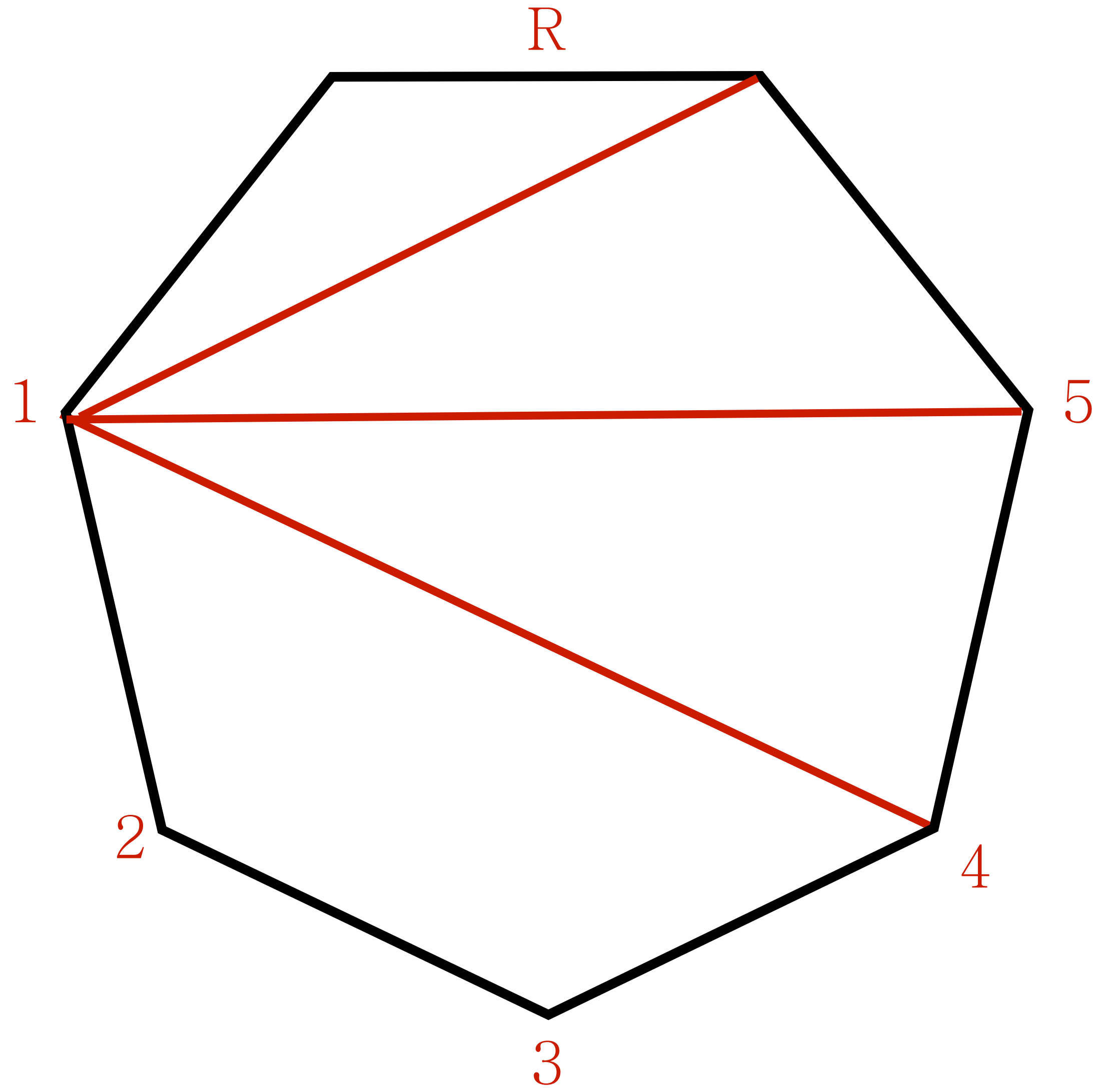

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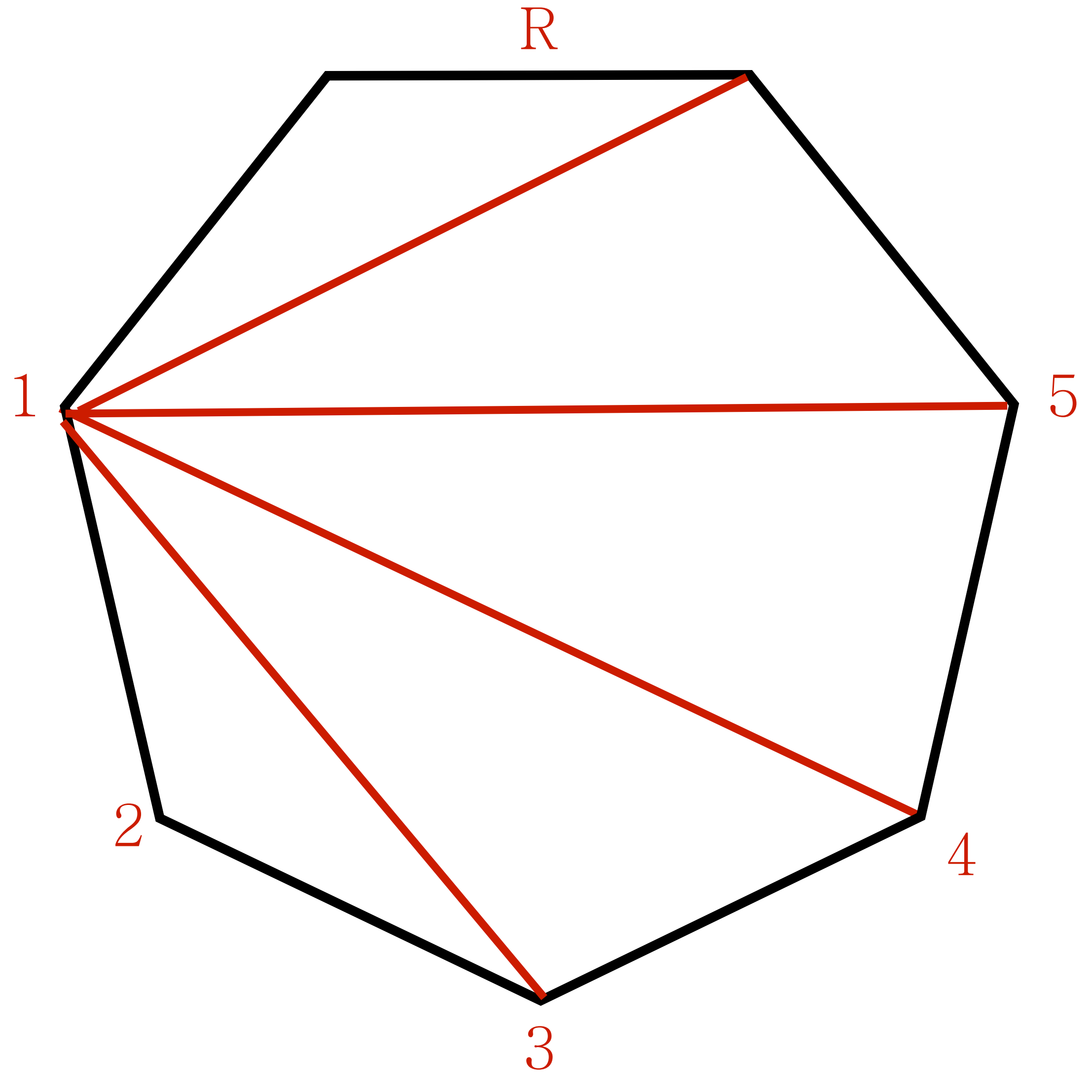

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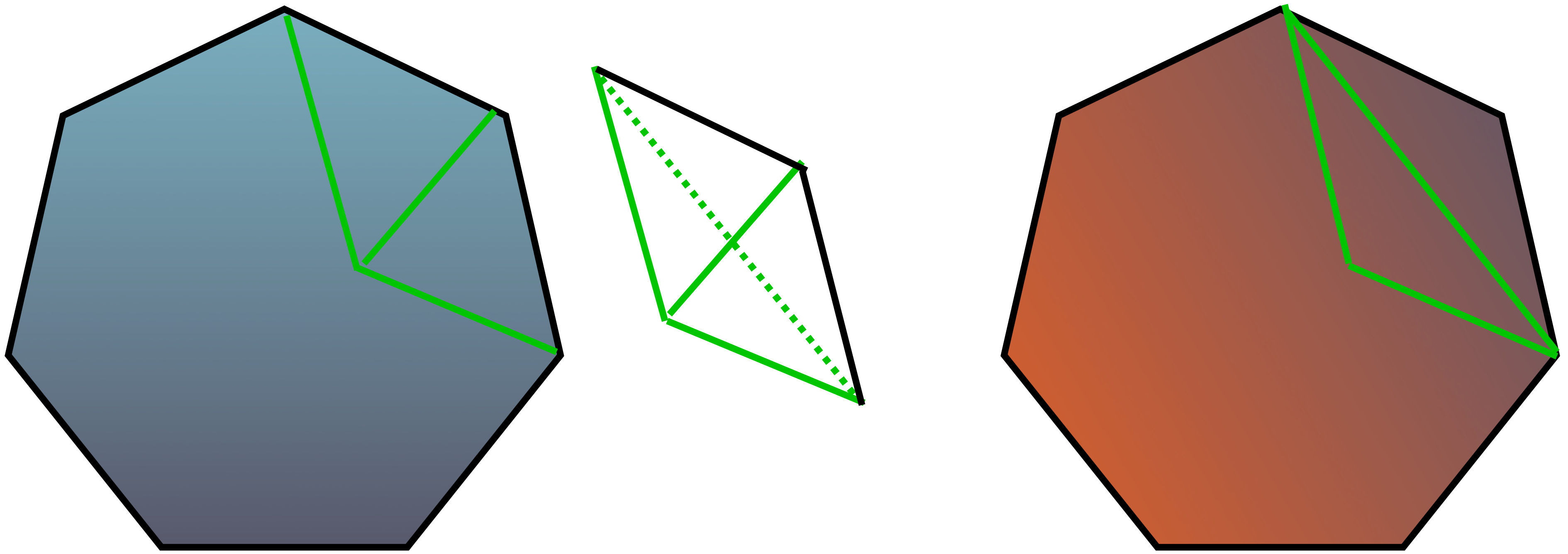
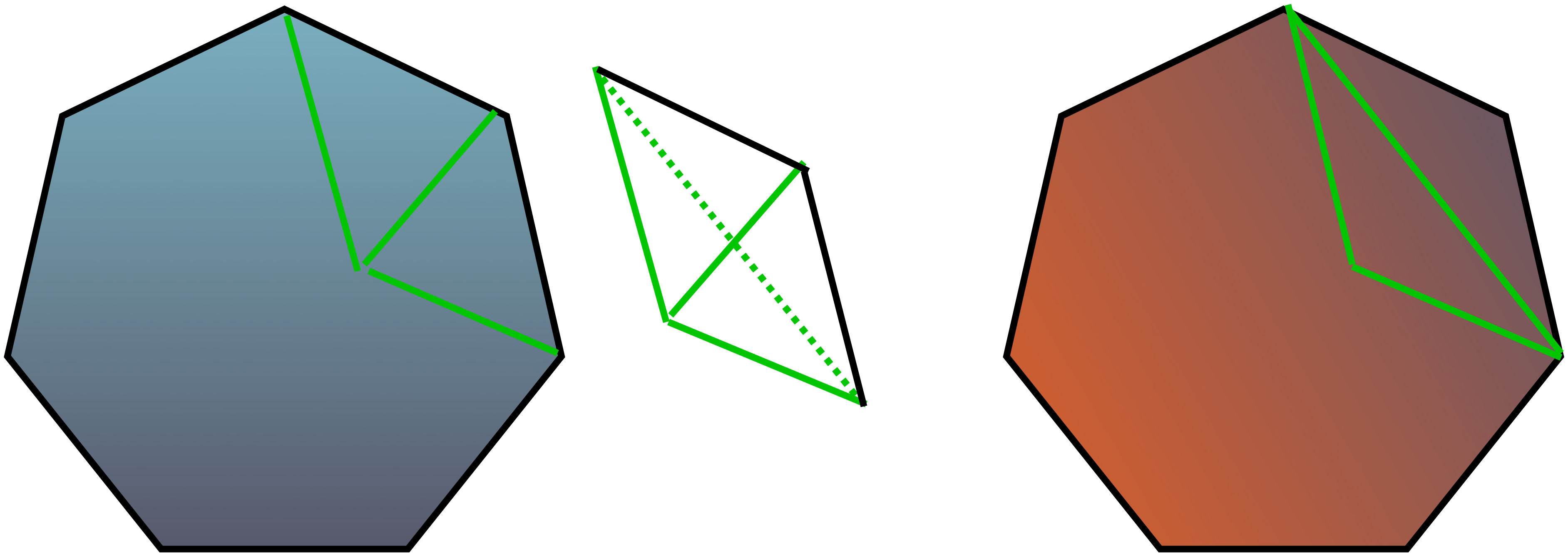


DEFINE: $t(n+2)$ = min # of tetrahedra in a triangulation of a ball interpolating between two triangulations.

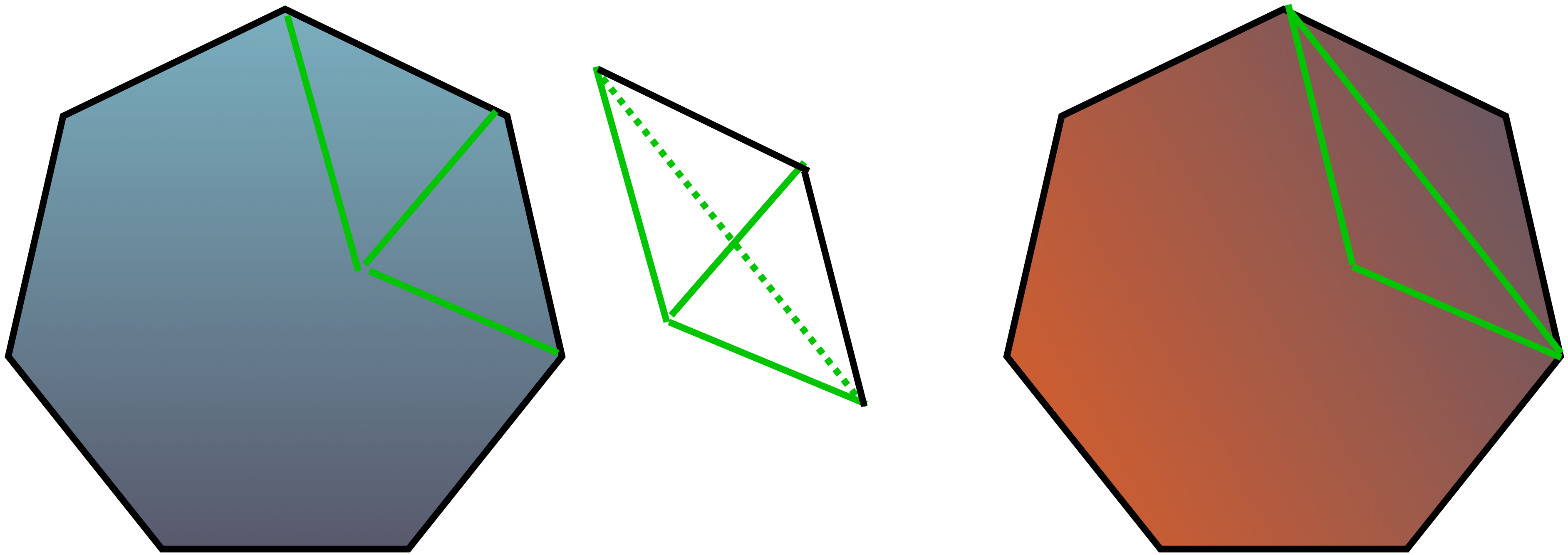
THEN: $t(n+2) \leq d(n)$

GEOMETRY



NEED TO FIND: $K(n+2) \leq t(n+2)$

GEOMETRY

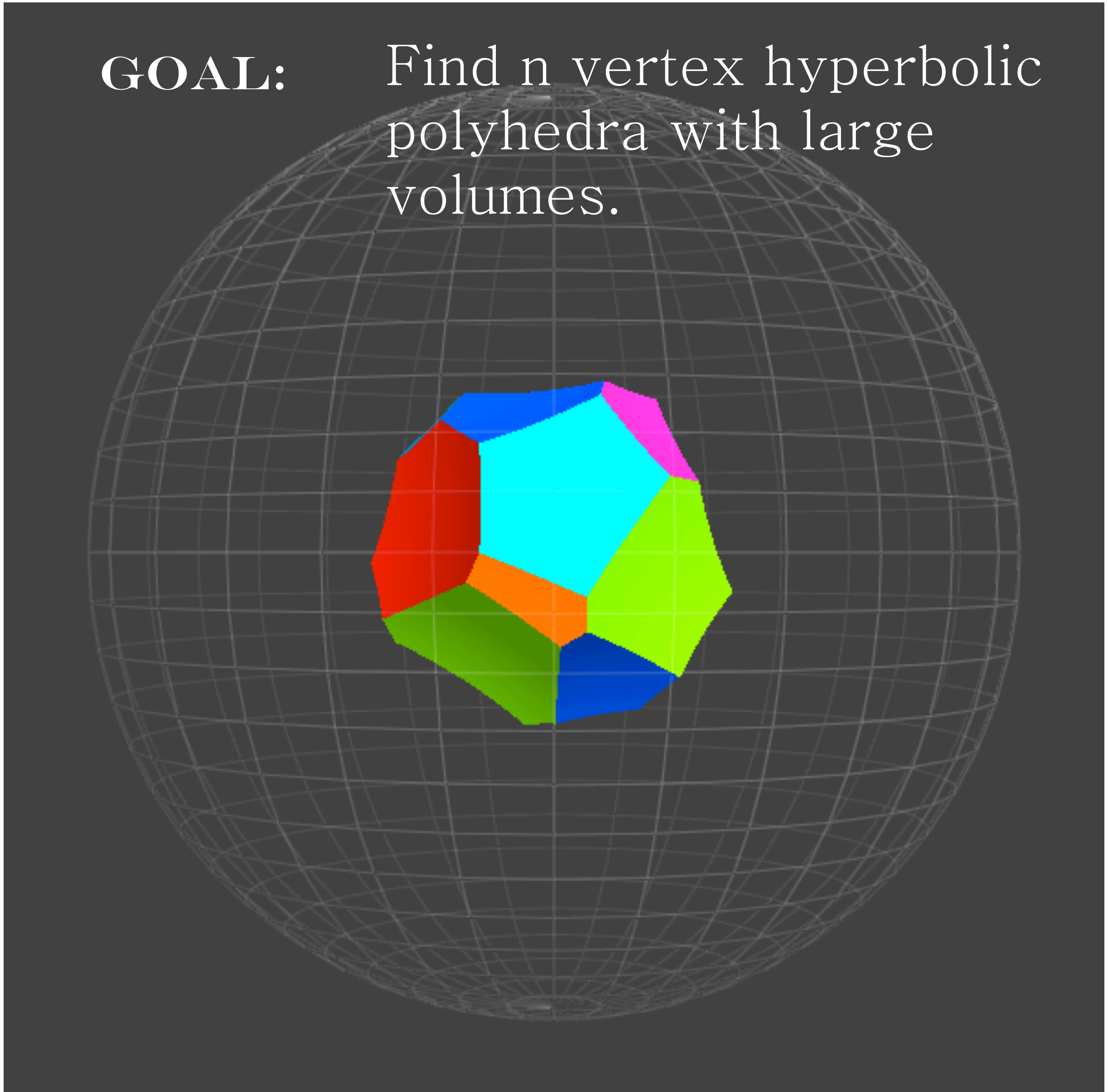


IDEA: Put polyhedron P into hyperbolic space and show:

vol(P)/V $\leq$ # tetra interpolating.

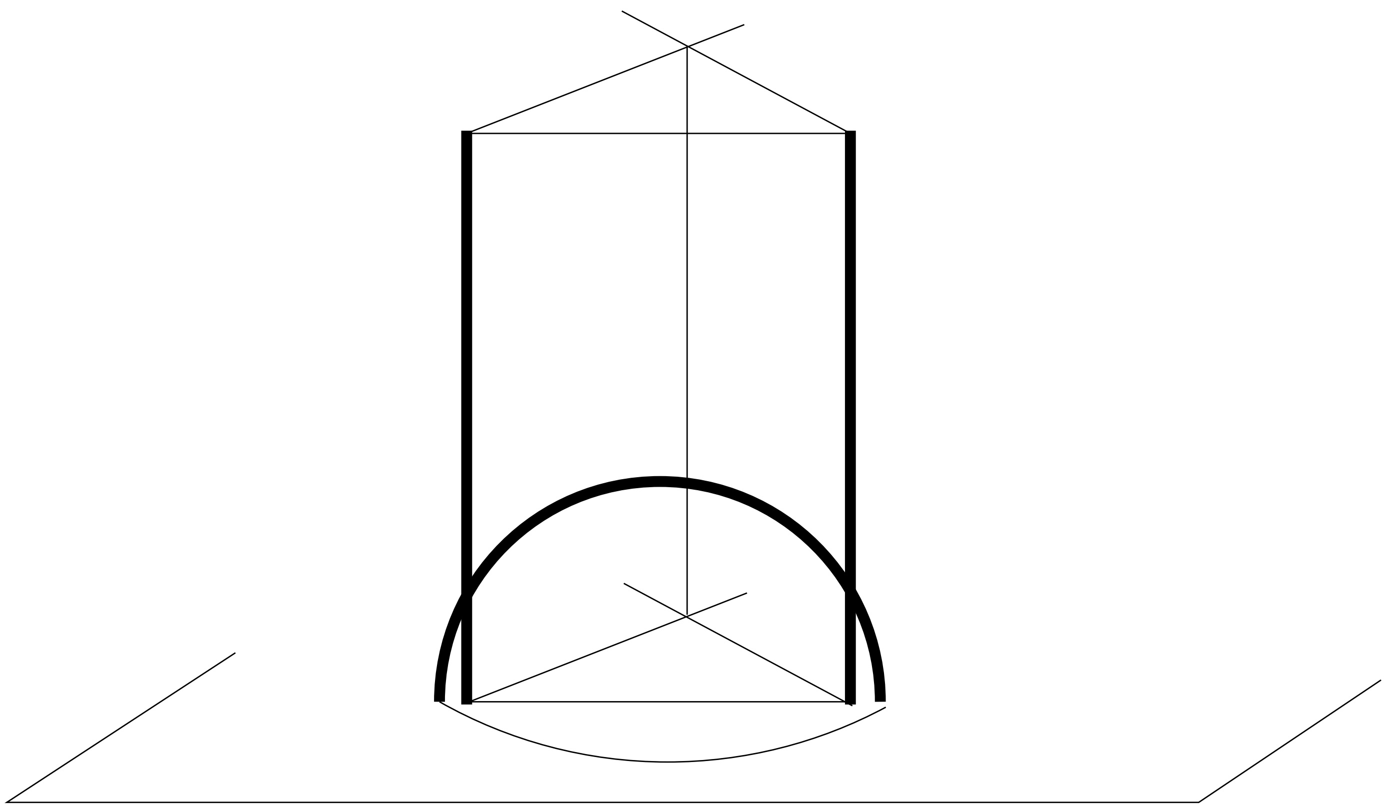
where V is the max volume of a tetrahedron in hyperbolic space $\approx$ 1.0149...

GEOMETRY


GOAL:
Find n vertex hyperbolic polyhedra with large volumes.

GEOMETRY

SHOW: $$t(n+2) \geq 2(n+2) - \text{``}\sqrt{n}\text{''}$$



GEOMETRY

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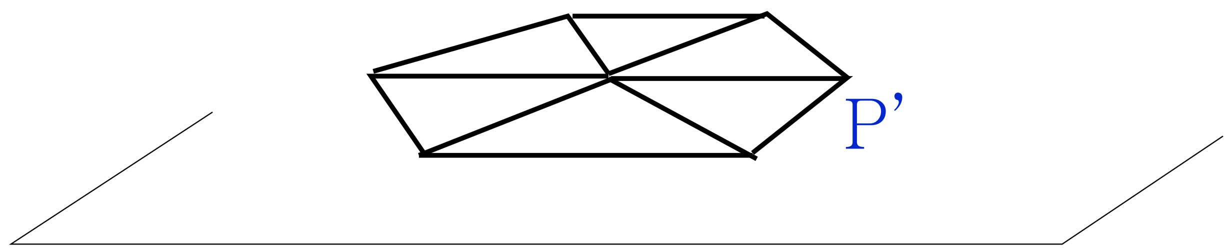
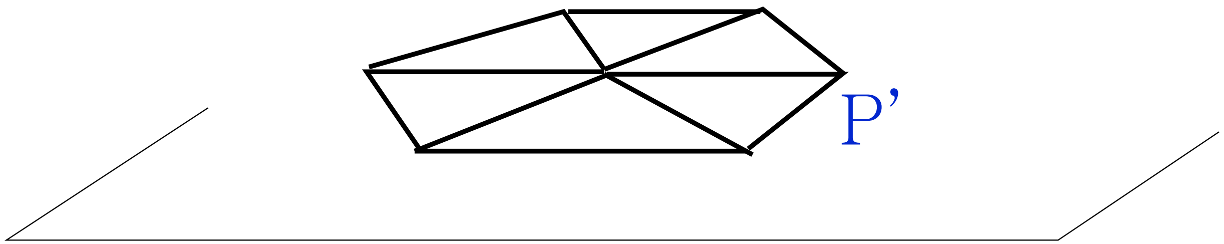


GEOMETRY

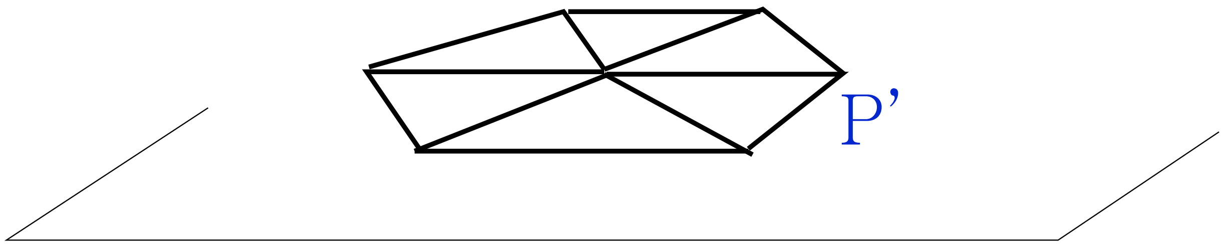
SHOW: $t(n+2) \geq 2(n+2) -$ "$\sqrt{n}$"



If P' is a hexagon with k edges per side there are $6k^2$ triangles and $3k^2 + 3k + 1$ vertices.

GEOMETRY

SHOW: $t(n+2) \geq 2(n+2) - \text{“}\sqrt{n}\text{”}$



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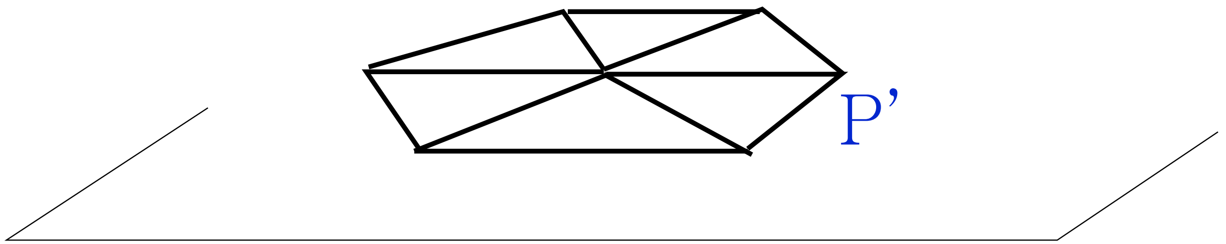
Then triangulation of P has $6k^2$ tetrahedra and $3k^2 + 3k + 2$ vertices.

GEOMETRY

SHOW: $t(n+2) \geq 2(n+2) - ``\sqrt{n}"$



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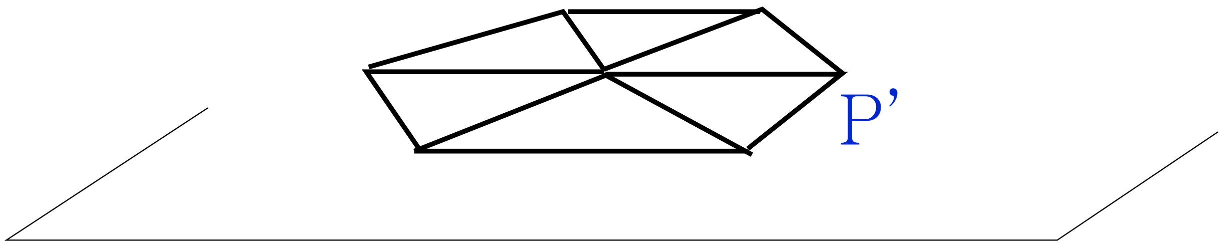
$$t(n+2) \geq 6k^2$$

GEOMETRY

SHOW: $t(n+2) \geq 2(n+2) - \text{``}\sqrt{n}\text{''}$



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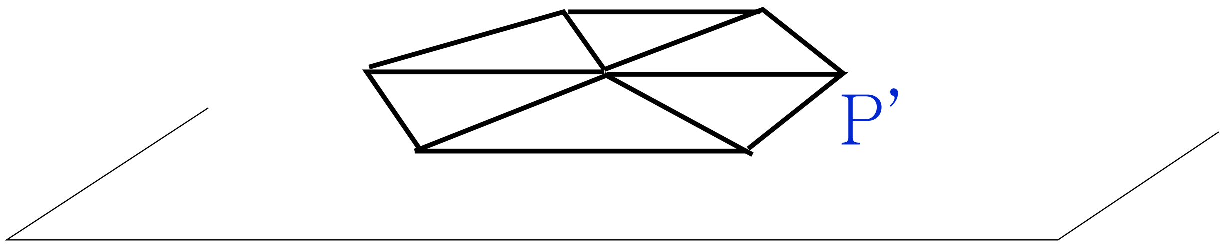
Then triangulation of P has $6k^2$ tetrahedra and $3k^2 + 3k + 2$ vertices.

$$t(n+2) \geq 6k^2$$
$$= 2(3k^2 + 3k + 2) - 6k - 4$$

GEOMETRY

SHOW: $t(n+2) \geq 2(n+2) - \text{“}\sqrt{n}\text{”}$



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Then triangulation of P has $6k^2$ tetrahedra and $3k^2 + 3k + 2$ vertices.

$$\begin{aligned} t(n+2) &\geq 6k^2 \\ &= 2(3k^2 + 3k + 2) - 6k - 4 \\ &\geq 2(n+2) - \text{“}\sqrt{n}\text{”} \end{aligned}$$