MA 398	**Geometry**	**Spring 2012**
Section A	**MWF 10 - 10:50**	**Keyes 103**

Professor:	Scott Taylor
Office Hours:	MWF 1 - 3. Tues 11 - 1 *and by appointment!*
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Prerequisites:	MA 122 or permission of instructor
Text:	Francis Bonahon, *Low-Dimensional Geometry* Richard Evan Schwartz, *Mostly Surfaces*

The Course:

MA 398: Geometry, explores the notion of "geometry" by introducing metric spaces and the most important 2-dimensional geometries: Euclidean, Spherical, and Hyperbolic. These geometries will be investigated using calculus and linear algebra. The main result that will be proven in this course is that every compact 2-dimensional surface admits either a euclidean, spherical, or hyperbolic geometry. As time allows, we will also investigate ways of parameterizing the different geometries admitted by a surface and examples of 3-dimensional spaces admitting a hyperbolic geometry.

Objectives for increasing mathematical maturity:

- Combine algebraic, analytic, and geometric modes of thought to systematically investigate different types of geometry.
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics.
- Write proofs in conventional mathematical style

Major Course Content Objectives

- Understand the basic outline of the historical progression of understanding of "geometry" and its influence on culture.
- Understand the axioms for euclidean, hyperbolic, spherical, and metric spaces.
- Understand the relationship between notions of distance and path length.
- Apply calculus in different geometric settings

- Understand and prove major differences between euclidean, spherical, and hyperbolic geometry
- Formalize intuitive geometric notions such as "gluing", "area", and "geodesic" and verify their properties in different geometric settings.
- Understand the relationship between 2-dimensional geometries and linear algebra.
- Prove that every closed surface supports a eucidean, hyperbolic, or spherical geometry.
- As time allows, investigate one or more of the following concepts: 3-dimensional geometries, Teichmüller space, Kleinian groups

Attendance: I value your involvement in the class, therefore class attendance is mandatory. Absence for official Colby activities requires prior approval. Absence for religious reasons will be considered excused if the policy in the college catalogue is followed. I reserve the right to take attendance. More than 3 recorded unexcused absences will result in the reduction by 1/3 of the final course grade. Excessive tardiness or early departure may also result in such a reduction.

Computing Resources: You may wish to use the software to complete some homework problems throughout the semester. *Mathematica* is available on many computers at Colby, including the computers in Mudd 415 and Olin 323. From Colby's fileserver you may also download and install *Mathematica* on your personal computer for use while on campus. Details will be distributed later.

You may also wish to make use of two other pieces of software, both of which are free:

- *WolframAlpha* (wolframalpha.com) is a web-based computational knowledge engine. It can be a bit tricky to use, so you will need think about whatever response it gives you. Another similar tool from the Wolfram people is available at integrals.wolfram.com
- *Grapher* is software which comes bundled with every modern Macintosh. (It can be found under Utilities). It is very easy to use and can draw almost every imaginable type of 2 and 3 dimensional graph. (Including solutions to differential equations and vector fields.) I highly recommend it – it is particularly easy to use.

You are allowed to use software to solve any integral you encounter in this course.

Academic Honesty: All work in this course must be your own and you should always be prepared to explain or defend it. You are encouraged to work with other students on the homework assignments, but you may not copy another student's work. On the take-home exams you may use your own texts and notes, but may not discuss the exam with others (except the instructor) and may not use other resources, in print or online. On the final exam you may not use any notes or books and may not discuss the exam with anyone except the instructor. Please read the section on Academic Honesty in the catalogue (pg 43).

Evaluation: The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

25%	Homework
5%	Participation
5%	Reading/Colloquium Responses
10%	Term Project
15/20%	Exam 1
15/20%	Exam 2
20%	Final Exam.

The maximum of your scores for Exams 1 and 2 will count at 20% of your grade. The minimum of your scores on Exams 1 and 2 will count as 15% of your grade.

Caveat: earning less than 50% on each of the three exams will result in a grade of "F" for the course.

Course letter grades will be assigned (subject to the above caveat) according to the following scale. Any curve will be determined at the end of the course according to the discretion of the instructor. An A+ may be awarded to students who do exceptionally well and who demonstrate an exceptional interest in the course.

93 – 100 % A	90 – 93 % A-	87 – 90 % B+	83 – 87% B
80 – 83 % B-	77 – 80 % C+	73 – 77 % C	70 – 73% C-
67 – 70 % D+	63 – 67 % D	60 – 63 % D-	below 60% F

Homework: Homework is the most important part of this course. It is where you wrestle with new ideas, generate creative solutions, and convey your thoughts to others. You are expected to write complete, careful, and correct proofs. You should start the homework early, some problems will require multiple attempts and careful thought. If you are having substantial difficulty with a particular problem or the entire homework set you should

email me or come to office hours. I am eager to help you! You are encouraged to work with a partner on the homework, but all work should be your own. In other words, you may discuss particular problems but you may not copy someone else's solution. Doing so violates academic honesty.

Structure and Schedule: Homework will be assigned and collected each class period. It will always be posted on the course webpage. You are responsible for checking the webpage. If no homework assignment is posted, you should refresh the webpage on your browser and, if that doesn't work, email me to let me know. In the special circumstance that there is no new homework, the webpage will make note of that. If you will not be in class on the day that homework is due you should arrange to turn it in at my office or to have a friend bring it to class. Late homework may be penalized.

Each homework set will consist of one or two "Huts", one or two "Houses", and one "Cathedral". The "Huts" will be problems that simply test understanding of concepts from class or the reading and should be fairly routine. Often they will simply be calculations. The "Houses" are problems that require synthesis, creativity and ingenuity. They are the central point of the course and are designed to improve your geometrical intuition, mathematical maturity, and proof-writing. The "Cathedrals" are problems that require a lot of creative effort. "Cathedral" problems are worth a relatively small part of the grade – if you can't solve them you shouldn't feel bad. However, to get a stellar grade in the course you should successfully solve a fair number of them over the semester.

Grading and Rewrites: You will receive simply "credit" or "no-credit" for doing Hut problems. House and Cathedral problems will be thoroughly graded for both mathematical correctness and completeness and clarity of the proof. Any House or Cathedral problem for which you turned in an (attempted) solution on time may be rewritten within one week of its return to you. Rewrites are graded using the same criteria as the original and you have the potential to earn full credit on such problems.

Because there will be a lot of paper being passed back and forth between us, you must follow these guidelines for homework:

- Each assignment must have your full name, the date it is due, and the assignment number on each page of the assignment.
- Problems must be turned in in the order they are assigned.
- All solutions must be turned in on 3-hole punched paper and placed into the "official" homework binder.

- Each House and Cathedral problem must be written on its own 2-sided sheet of paper. You are encouraged not to turn in your first draft of any of these problems.
- Rewrites must be turned in with the original solution. No credit will be given for rewrites without the original attached. It must be clear exactly which problems from which assignments are being rewritten.
- Homework must be very neat. This means: no messy scratchwork, no cramped writing, no huge eraser marks. If these guidelines are not followed you may be penalized. If you are incapable of writing neatly, you should type your solutions. LaTex is the most popular mathematical typesetting software, but you may also use software like *Scientific Word* or *Mathematica*.

Proof Expectations: It is expected that every graded proof be written in conventional mathematical style. Each statement should follow logically and naturally from the previous ones. In general, your work is your answer. It is possible for a proof to be substantially correct but not to receive full credit because the writing was not up to standard. The audience you should keep in mind is an imaginary classmate who is not quite as "bright" as yourself. The danger in communicating mathematics is always assuming your audience knows more than they do!

Project: This semester, our class is responsible for the April Science Library exhibit. Details will be distributed separately, but essentially each person in the class is responsible for contributing two "exhibit cards" to the exhibit and contributing to a jointly-compiled handout. The project will take a fair amount of time and will require independent investigation, exploration and writing. Completing the project will help improve your ability to convey mathematical concepts to non-mathematicians and will enable to encounter geometric ideas that cannot be fit into the regular content of the course.

Participation: I expect that you will volunteer answers to questions posed to the class as a whole and that you will be prepared to make a contribution to the discussion if specifically called upon. In the event that I call on you specifically, you do not necessarily need to be able to answer the question I ask, but you should be prepared to give a partial answer, an intelligent guess, or to ask a relevant question. I expect that you will have attempted each day's homework prior to class.

Readings and Colloquium Responses: You are required to do four of the following activities:

(1) Attend a Mathematics/Statistics Colloquium and write a page summarizing and reacting to the lecture.
(2) Read one of the distributed articles and write a page summarizing and reacting to the article.

Exams: There will be two take-home exams and a final exam. Each exam is cumulative, although the final exam is "more cumulative". Exams will be designed to test your understanding of the course material and your ability to devise original proofs. You must understand, and communicate your understanding of the material. You may not discuss the exam with other people and you may not use any resource other than your notes and textbooks. Here is the schedule of exams:

(1) Exam 1: Distributed on Friday, Feb. 24 and due by 12 PM on Tuesday, Feb. 28.
(2) Exam 2: Distributed on Friday, April 13 and due by 12 PM on Tuesday, Feb. 17.
(3) Final Exam: Exam period 4; Thursday, May 10; 9 AM - 12 PM.

How to succeed in this course:

Geometry is an intense 300-level mathematics course. The course, therefore, expects a level of sophistication and mathematical maturity beyond that expected by introductory calculus courses. In particular: you are responsible for your own learning and the professor is responsible for assisting you in your quest. In order to truly understand vector calculus you need to be willing to do computations and you need to be willing to wrestle with ideas. The following advice is meant to provide suggestions on get more out of the class and to improve your grades:

(1) *Do some math everyday.* Really.

    Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The daily homework problems are intended to help you in your daily practice. But you should spend additional time each day working on the weekly homework, reading the text, and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that's 6-9 hours per week of homework and studying.

(2) *Participate in class.* Yes, you.

    Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on.

The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a questions is a great way to learn the right answer. Make an effort to connect each class's activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch.

(3) *Read the textbook.*

The lectures and the textbook will often present slightly different views on the same material. The examples in the text will (for the most part) differ from the example presented in class. As you read the text, relate what the text does to what happened in class where is it the same, where is it different? If it differs, which approach do you like better what are the pros and cons of each?

(4) *Form a study group.*

Introduce yourself to other people in the class and meet up outside of class to study and work on homework. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other. Calculus-After-Hours is for 100-level calculus students, but it makes the math department a happening place – it's a great place to meet up with your classmates.

(5) *Write your own problems.*

As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems what makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.

(6) *Be curious.* We will be spending a lot of time, exploring the theoretical mathematical ideas. Be curious about why geometry (or a geometry) works the way it does. Ask "why". Try to find applications of what we're learning. Try to invent your own geometry. Schedule time for curiosity. Block off 2 hours every day for Geometry. Any time not spent working on homework, spend studying and playing with geometric concepts. Come up with your own research questions. Computers compute; humans ask creative questions.

(7) *Visit me in my office.*

I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it's not my office hours. If I cant chat, Ill let you know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event.

(8) *Spread the studying out over the semester.*

If you do math everyday, as suggested above, you wont have to cram for exams. Youll be able to sleep and, consequently, to think. You'll be happier and more relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the snow and the spring. You don't need to pull all-nighters.