

## MA 398 Homework 5: Scratching the Surface

This homework is intended to be lighter than usual. The goal is to get you used to working with surfaces.

### 1. HUTS

These problems are intended to give you some practice with basic concepts. They will often involve calculation, rarely involve new ideas, and won't be graded. However, your answers will be collected!

In class we discussed ways of gluing polygons to obtain the torus and the genus 2 surface. For these problems you can work pretty informally – draw pictures, don't write equations.

- (1) Find a way of gluing the edges of a regular hexagon to obtain a torus.
- (2) Find another way of gluing polygons together to get a genus 2 surface.

### 2. CATHEDRALS

This problem gets you to think about a particularly important, albeit simple, gluing operation: Let  $P = \{(x, y) \in \mathbb{R}^2 : -1 \leq x \leq 1\}$ . Notice that  $\partial P$  consists of the lines  $x = -1$  and  $x = 1$ . For each  $r \in \mathbb{R}$ , let

$$\phi_r(-1, y) = (1, y + r).$$

- (1) Draw a picture and describe in words, the surfaces  $S_r$  resulting from gluing the sides of  $P$  using  $\phi_r$ . Also state how the surfaces differ for different values of  $r$ .
- (2) Prove (using the euclidean polygon theorem) that for each  $r$ ,  $S_r$  is locally isometric to  $\mathbb{R}^2$  with the usual metric.
- (3) Prove, to the best of your ability, that  $S_0$  has a closed curve which is locally length minimizing, but for  $r \neq 0$ ,  $S_r$  does not.
- (4) (Bonus) Prove that if  $r \neq s$  then  $S_r$  is not isometric to  $S_s$ .